On global warming

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1 Fitting the increase in CO_2 to the temperature curve

The ongoing debate about global warming prompted me to have a look at the physics myself, and see if I could make some sense of it. I have to say that I came to this rather as a sceptic because I do not believe that it is feasible to model the climate of the planet with any degree of confidence. The number of relevant variables is just too great. It is well-known that the weather is a chaotic phenomenon, and a look at the temperature variations indicates that the yearly trend is equally random. On the other hand, NASA's chart of the world average temperatures does show an undeniably upward trend over the last 130 years.

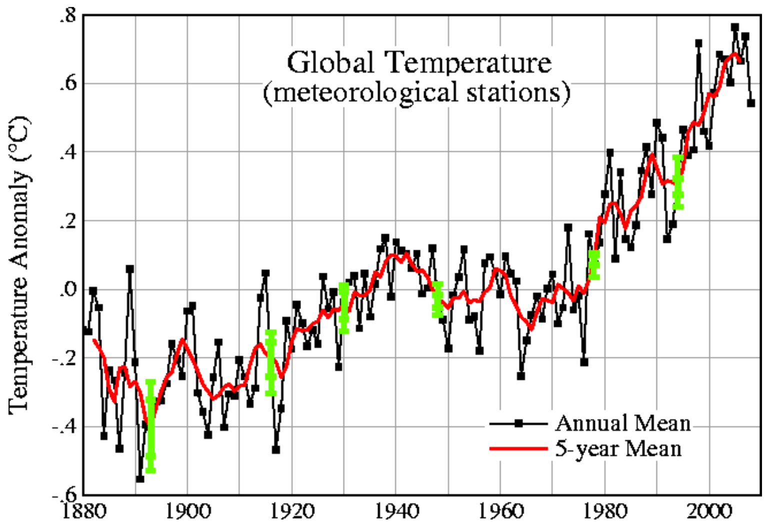

Figure 1. *World average temperature since 1880 according to NASA's Goddard Institute for Space Studies.*

It could therefore be that there is indeed a simple explanation for this trend even if one cannot predict the random fluctuations superimposed on it. However, in that case there should surely be a simple model explaining the phenomenon. If one wants to explain magnetism, for example, one does not begin by writing an elaborate computer program involving all the atoms of the material and the forces between them. Even if this were successful, one would still not have obtained an understanding of the phenomenon. One needs a simple model which captures the essential features and explains the origin of the particular phenomenon, i.e. magnetism.

To see if the increase in CO_2 could in principle explain the observed increase in world mean temperature, we need to understand how CO_2 contributes to the heating of the atmosphere. As is well-known, incoming solar radiation is absorbed by the Earth and then re-radiated at longer wavelengths, essentially the infrared. This is easily calculated approximately. One just needs a few astronomical data. The incoming solar flux is 1367 $\mathrm{J/m^2, sec}$, and the radius of the Earth is $R = 6357$ km, so the area facing the Sun is $\pi R^2 = 1.27 \times 10^{14}\,\mathrm{m}^2$, so the total energy budget is 1.735×10^{17} Watt. However, the *albedo* of the Earth is about 30%, which means that that fraction of the incoming radiation is reflected back into space (off clouds and the Earth's surface). That leaves $0.7 \times 1.735 \times 10^{17} = 1.21 \times 10^{17}$ W which is absorbed. By energy balance, this amount of energy has to be radiated back into space in the form of infrared radiation. If we assume that the Earth acts roughly as a black body, the radiation follows the Stefan-Boltzmann law [4, 8]

$$I = \sigma T^4, \text{ where } \sigma = 5.67 \times 10^{-8}\,\mathrm{W/m^2, K^4} \tag{1.1}$$

according to Planck. This yields

$$T^4 = \frac{1.21 \times 10^{17}}{\sigma \cdot 4\pi R^2} = \frac{0.7 \times 1367}{4 \cdot 5.67 \times 10^{-8}} = 4.2 \times 10^9\,\mathrm{K}^4.$$

Hence $T = 255$ K. This is the effective temperature at which the Earth radiates. However, the average surface temperature is known to be $15\,^0\mathrm{C} = 288\,\mathrm{K}$. Largely, this is due to the atmosphere lying like a blanket over the Earth. Indeed, the temperature varies with height as any climber knows. The atmosphere is heated by absorbing radiation from the Earth (and also from the Sun directly, but this mainly in the upper regions by oxygen and ozone, where the density is very low). The largest contributor to the absorption of infrared radiation is water vapour. However, CO_2 also contributes because it absorbs at different wavelengths.

The absorption spectra of individual molecules consist of many sharp peaks (spectral lines), but these are broadened in gases essentially due to two effects:

1. collision broadening, and
2. Doppler broadening.

Collision broadening is due to collisions between the molecules of a gas, which disturbs their energy levels; Doppler broadening is due to the Doppler effect: the random thermal motion of the molecules results in an apparent shift in the frequency of the radiation absorbed. At normal atmospheric densities, collision broadening dominates, but at the low densities in the upper atmosphere, this effect decreases in importance and Doppler broadening takes over. Whereas Doppler broadening has a Gaussian profile, collision broadening is roughly Lorentzian in shape:

$$g_{\mathrm{Doppler}}(\nu) = C \exp[-r(\nu - \nu_0)^2], \tag{1.2}$$

versus

$$g_{\mathrm{coll}}(\nu) = \frac{C}{a^2 + (\nu - \nu_0)^2}. \tag{1.3}$$

Here ν is the frequency, ν_0 is the centre of the spectral line and C, r and a are constants. The two different shapes are illustrated in Figure 2:

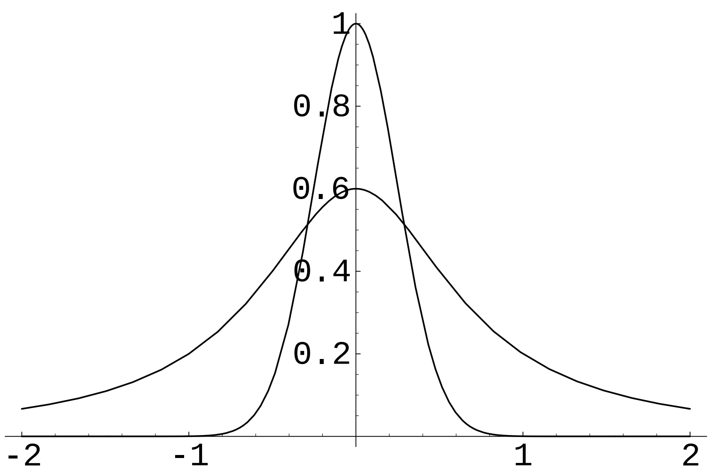

Figure 2. *Lines shapes for Doppler and collision broadening.*

Since most of the absorption takes place in the lower layers of the atmosphere, we can therefore assume that collision broadening dominates, and the lines shape is roughly Lorentzian. Note that this shape has much fatter tails, which is important in the following.

The spectral line basically indicates the probability that a given molecule absorbs light of a given frequency. As light traverses the atmosphere it can be absorbed by the successive molecules that it encounters, so by the law of compound interest, the intensity of the light escaping is exponentially decreasing in the density (or concentration c) of the molecules in the atmosphere as well as height of the spectral line at the given frequency. The absorbed fraction of the radiation is thus

$$f(\nu) = 1 - \exp[-\lambda c g_{\mathrm{coll}}(\nu)]. \tag{1.4}$$

For a single line the result looks as follows:

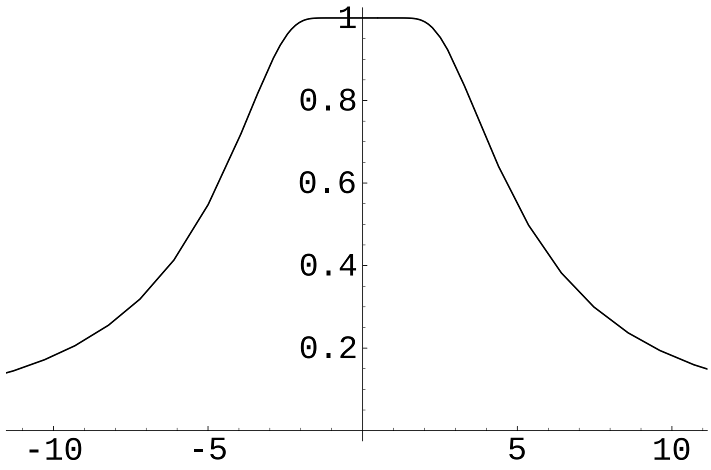

Figure 3. *Absorbed fraction for a broadened line.*
The total absorption is therefore given by

$$\kappa(c) = \int_{\nu_1}^{\nu_2} \{1 - \exp[-\lambda c g_{\mathrm{coll}}(\nu)]\} \, d\nu, \tag{1.5}$$

where the integration is over the width of the spectral band $[\nu_1, \nu_2]$. This behaves, for not-too-large concentrations, like

$$\kappa_{\mathrm{approx}}(c) = \kappa_0 + \mu\sqrt{c} \tag{1.6}$$

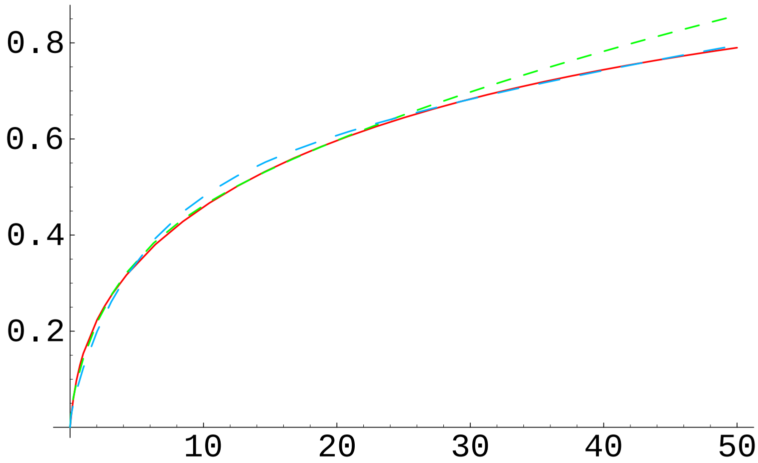

Figure 4. *Total absorption as a function of the concentration.*

The figure also shows two curves of the form (1.6). One approximates $\kappa(c)$ better for smaller c; the other for larger values of c. The $\sqrt{c}$ behaviour is not difficult to understand. For $c \gg a^2$ and $\nu_0 \gg \sqrt{c}$,

$$\begin{aligned} \int_{-\nu_0}^{\nu_0} \left\{1 - \exp\left(-\frac{c}{a^2+\nu^2}\right)\right\} d\nu &\approx \int_{-\nu_0}^{\nu_0} \left(1 - e^{-c/\nu^2}\right) d\nu \\ &= 2\sqrt{c}\int_0^{\nu_0/\sqrt{c}} (1 - e^{-1/u^2}) du \\ &\approx 2\sqrt{c}\int_0^{\infty} (1 - e^{-1/u^2}) du - 2\sqrt{c}\int_{\nu_0/\sqrt{c}}^{\infty} \frac{du}{u^2} \\ &= 2\left(1.772 - \frac{\sqrt{c}}{\nu_0}\right)\sqrt{c}. \end{aligned} \tag{1.7}$$

It now seems reasonable to fit a curve of the form

$$T = T_0 + k\sqrt{c} \tag{1.8}$$

to the temperature graph, using the measured values for the CO_2 concentration: see [19].

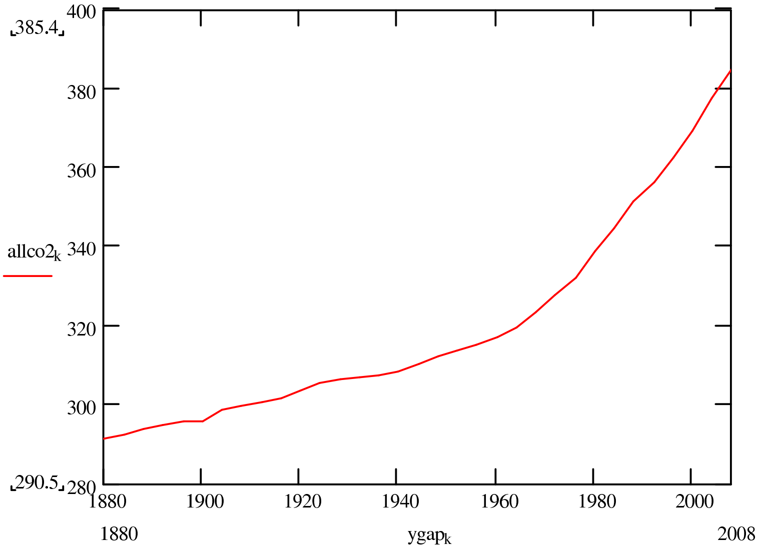

Figure 5. *Measured CO_2 concentration.*
Adjusting the two arbitrary constants (T_1 and k), the result is remarkable:

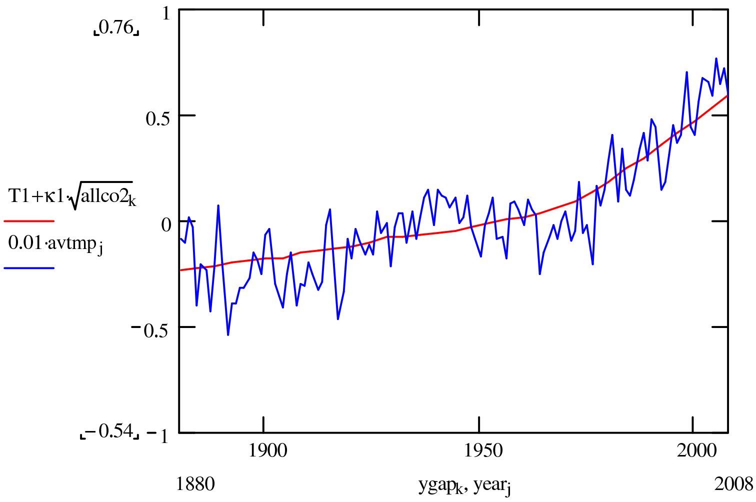

Figure 6. *Comparison of the actual world mean temperature with a best fit dependence on the CO_2 concentration.*

Of course, given this model it is easy to extrapolate in time given a scenario for the future increase in the CO_2 concentration. In Figure 7 we compare the consequences of a linear increase with that of an exponential increase. It shows that the former leads to an estimated increase in temperature by the year 2100 of about 2 ^{0}C, while the latter leads to an increase of about 4 ^{0}C.

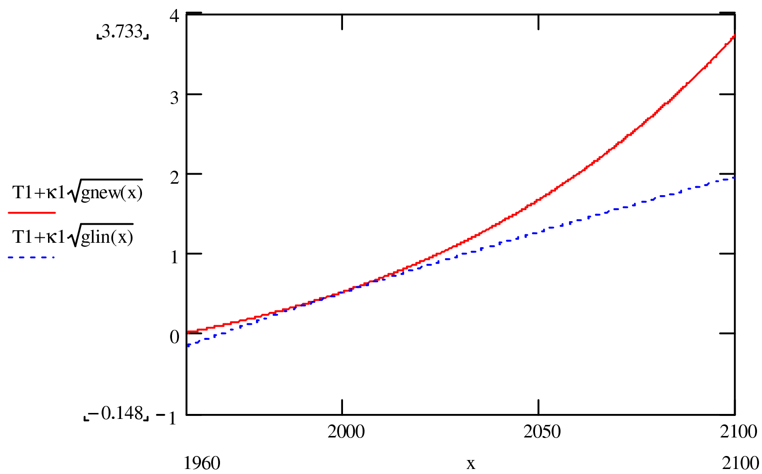

Figure 7. *Estimated temperature increase by 2100 based on two scenarios: a linear in crease in CO_2 and an exponential increase resp.*

It is also of interest to subtract the computed CO_2 background from the actual measured average temperature:

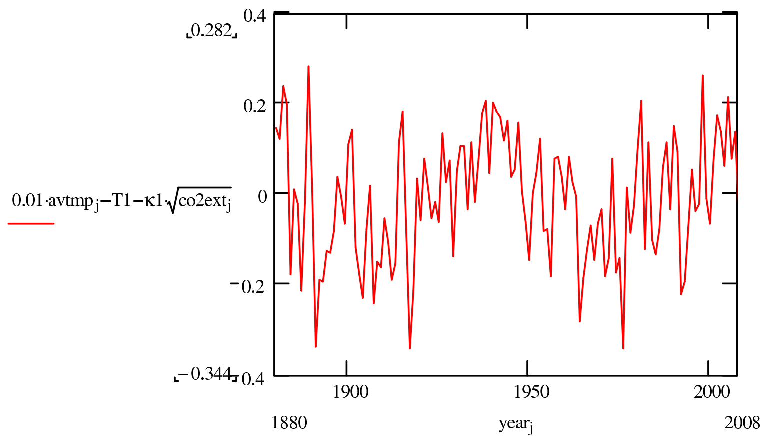

Figure 8. *Residual temperature fluctuations after subtraction of the presumed CO_2 contribution.*

There is a curious apparent periodicity of roughly 60 years in this graph, for which I have no explanation.

2 Estimate of the CO_2 contribution to atmospheric warming

Clearly, the above analysis is unsatisfactory in that there are two adjustable parameters, and one might argue that it is easy to fit any increasing curve to another using two arbitrary parameters. In fact, this is not true. For example, a linear function also has two arbitrary parameters, and yet most increasing graphs are clearly not linear. I was therefore genuinely surprised to find such a good agreement.

All the same, one would like to have further theoretical confirmation to show that it is reasonable to expect CO_2 to make a contribution to the atmospheric temperature of the observed order of magnitude. Let us therefore examine more closely the mechanism of infrared light absorption. The relevant absorption spectra are shown below:

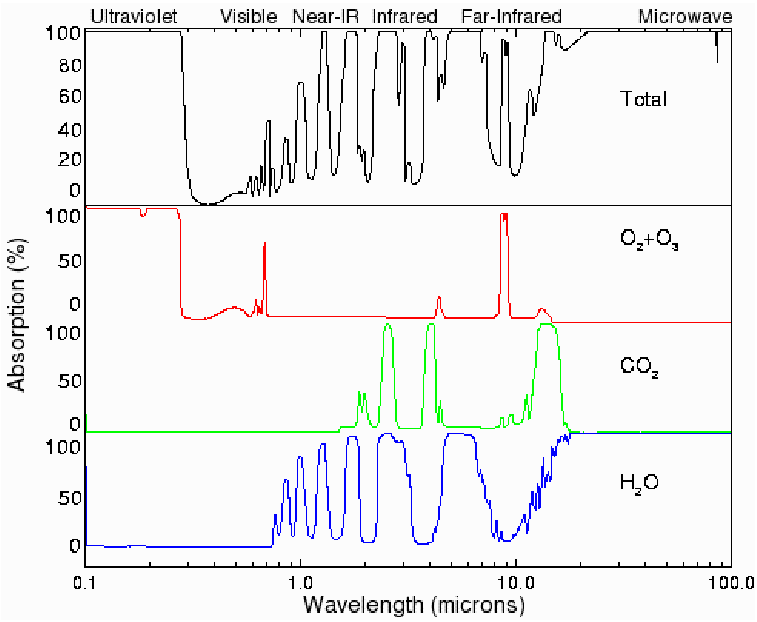

Figure 9. *Contributions to absorption of light in the atmosphere.*

The main contribution from CO_2 comes from its absorption band at a wavelength of 15 μm, which corresponds to a frequency of $\nu = \frac{c}{\lambda} = \frac{3 \cdot 10^8}{1.5 \cdot 10^{-5}} = 2.0 \cdot 10^{13}$ Hz. It has a shape $f(\nu)$ approximating a landscape with a central peak and two shoulders [13]:

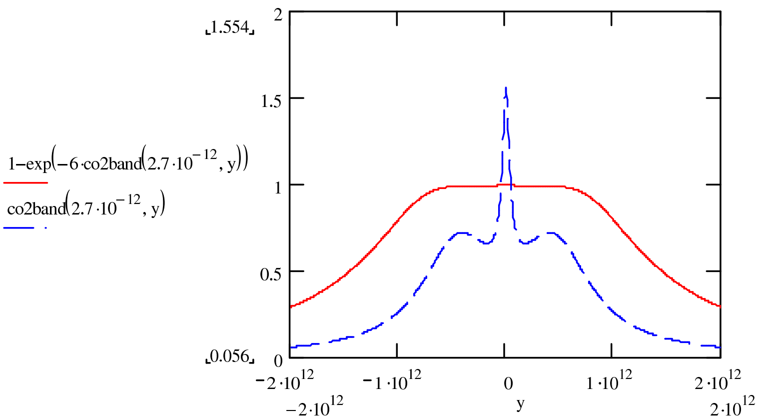

Figure 10. *Main absorption band of CO_2.*

The width of the band is approximately $2.8 \cdot 10^{12}$ Hz, which has to be compared with the Planck distribution of the radiation, given by

$$I(\nu, T) = \frac{2\pi h}{c^2} \frac{\nu^3}{\exp\left[\frac{h\nu}{kT}\right] - 1}. \tag{2.9}$$

Here $h = 6.626 \cdot 10^{-34}$ Js is Planck's constant, $c = 3 \cdot 10^8$ m/s is the speed of light, and $k = 1.38 \cdot 10^{-23}$ J/K is Boltzmann's constant, and of course $\pi = 3.1416$.

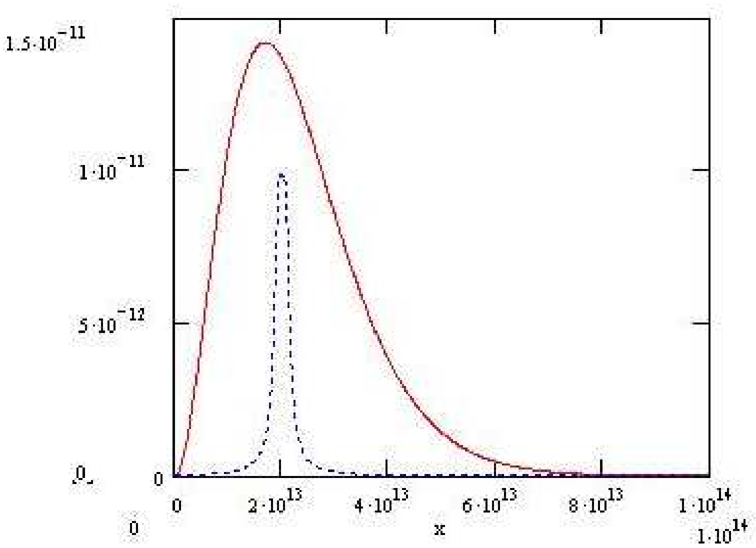

Figure 11. *The Planck distribution and the main CO_2 absorption band.*

Clearly, the absorption band is quite narrow, but on the other hand it is near the centre of the distribution. It seems that the three peaks in the spectrum are virtually saturated: see Figure 9. I therefore adjusted the scale λ of the CO_2 concentration such that this is the case. The resulting absorption $\kappa_c(\nu) = 1 - \exp[-\lambda c\, f(\nu)]$ is also plotted in Figure 10.

We can now approximate the contribution to the total absorption due to the main absorption band of CO_2:

$$\int_0^\infty I(\nu,T)\kappa_c(\nu)d\nu = 43\,\mathrm{W/m}^2, \tag{2.10}$$

taking $T = 288$K. This has to compared with the total energy density

$$\int_0^\infty I(\nu,T)d\nu = 389\,\mathrm{W/m}^2, \tag{2.11}$$

so the relative contribution is $\frac{43}{389} = 11\%$. In fact, this is probably an over-estimate as the low-frequency part of the absorption overlaps with that of H_2O.

To determine the increase in temperature corresponding to an increase in CO_2 concentration from 290 ppm to 385 ppm at present over the period 1880 until 2008, we assume detailed balance, i.e. the energy absorbed is radiated at a slightly higher temperature. Differentiating (1.1) we have:

$$\frac{\Delta I}{I} = 4\frac{\Delta T}{T}. \tag{2.12}$$

(Notice that this justifies taking the same linear relation between T and $\sqrt{c}$ as for the absorption $\kappa(c)$.) Because of the overlap with the H_2O absorption at low frequencies we only take into consideration the high-frequency half of the band. Moreover, we also introduce a cut-off at high frequencies of $3 \cdot 10^{13}$Hz because of overlap with an oxygen absorption band. The corresponding contribution is

$$\int_{2\cdot 10^{13}}^{3\cdot 10^{13}} I(\nu,T)\kappa_c(\nu)d\nu = 25\,\mathrm{W/m}^2. \tag{2.13}$$

Reducing the concentration to $\frac{290}{385}$ yields:

$$\int_{2\cdot 10^{13}}^{3\cdot 10^{13}} I(\nu,T)\kappa_{\frac{290}{385}c}(\nu)d\nu = 22\,\mathrm{W/m}^2. \tag{2.14}$$

Thus

$$\Delta T = \frac{1}{4}\frac{25-22}{389}255\,\mathrm{K} = 0.5\,\mathrm{K}. \tag{2.15}$$

This is in reasonable agreement with the observed increase. Of course, this calculation must not be taken too literally! Changing the scale of the concentration, which I adjusted somewhat arbitrarily, does make a difference. Moreover, the assumption of Lorentzian tails is also somewhat suspect. In particular, at higher altitudes collision broadening is reduced, resulting in a smaller contribution. On the other hand, I have ignored another absorption band at $7.5 \cdot 10^{13}$Hz. The point of the calculation is not so much to get a precise value but rather to show that it is not unreasonable to think that a substantial part of the increase in temperature is indeed due to the increase in CO_2.

3 Sea level rise

Potentially, one of the most serious consequences of global warming is sea level rise. One of the reasons for this rise is clearly thermal expansion. Let us try to make an estimate for this. Roughly the top 400-500 m of the ocean is well-mixed by convection due to wind force. Given an average thermal expansion coefficient of 2.7×10^{-4} per degree[1], this contributes about $450 \times 0.6 \times 2.7 \times 10^{-4} = 0.073\,\mathrm{m} = 7.3\,\mathrm{cm}$ over the last 50 years. In addition, there is a small contribution due to conduction to greater depth. This satisfies the heat equation

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c} \frac{\partial^2 T}{\partial x^2}, \tag{3.16}$$

where $\lambda = 0.59\,\mathrm{W/m, K}$ is the heat conduction coefficient, $\rho = 1025\,\mathrm{kg/m^3}$ is the density and $c = 3900\,\mathrm{J/kg, K}$ is the specific heat. Writing the diffusion constant $D = \frac{\lambda}{\rho c} = 1.48 \cdot 10^{-7}\,\mathrm{m^2/s}$, and approximating the temperature by $T \approx \gamma t^2$, the solution is

$$T(x,t) = \frac{8}{3\pi} \gamma \int_0^t ds\, s\sqrt{s}\, \frac{\exp[-\frac{x^2}{4D(t-s)}]}{\sqrt{t-s}}. \tag{3.17}$$

[1] Here I have calculated a rough average of the expansion coefficient as follows. Assuming the distribution of ocean water over the lattitude according to $\sin^2(\phi)$ (at northern latitudes the fraction of land is much higher than at the equator, but at southern latitudes it is less; of course a more accurate estimate is possible) and a temperature profile also according to $\sin^2(\phi)$ the average can be calculated as

$$\kappa_{\mathrm{av}} = \frac{2}{\pi} \int_0^\pi \kappa(36 \sin^2(\phi)) \sin^2(\phi) d\phi.$$

I took the temperature at the equator as 36 ^{0}C, perhaps an overestimate. $\kappa(t) \approx -0.610723 + 0.157876\, t - 0.00118677\, t^2$ according to the table in [2].

Therefore the contribution to sea level rise is

$$\begin{aligned} h(t) &= \int_0^\infty dx\,\frac{8}{3\pi}\gamma\int_0^t ds\,s\sqrt{s}\,\frac{\exp[-\frac{x^2}{4D(t-s)}]}{\sqrt{t-s}} \\ &= \frac{8}{3\pi}\gamma\sqrt{\pi D}\int_0^t ds\,s\sqrt{s} = \frac{16}{15\pi}\gamma\sqrt{\pi D}\,t^2\sqrt{t}. \end{aligned} \tag{3.18}$$

A reasonable quadratic fit to the temperature is

$$-0.3 + 5.5\cdot 10^{-4}\,t^2,$$

where t is measured in years since 1880. Taking $\gamma = 5.5\cdot 10^{-4}$ we obtain

$$h(t) = 1.9\cdot 10^{-8}\,t^2\sqrt{t}.$$

This yields a contribution of 3.7 mm since 1880. The current slope is calculated to be

$$\begin{aligned} &450\cdot 2.7\times 10^{-4}\cdot 2\gamma\,t + \frac{8}{3\pi}\sqrt{\pi D}\gamma t^{3/2} \\ = \;&450\cdot 2.7\times 10^{-4}\cdot 1.1\cdot 129 + \frac{5}{2}\cdot 1.9\cdot 10^{-8}(129)^{3/2} = 1.8\,\mathrm{mm/y}. \end{aligned}$$

The quadratic approximation perhaps slightly underestimates the slope for the later years. Using a linear approximation to the temperature since 1960, a similar calculation yields 1.9 mm per year. The agreement with Church [1] and Holgate [6] is rather good, given the rough estimates made. Holgate for example, estimates the current rate of sea level rise as 2 mm per year. This leaves only 0.1-0.2 mm/year unexplained, probably due to some glacier melting. Extrapolating the above trend until 2100 yields an additional rise of 22 cm and a slope of 2.9.

4 Conclusions and political remarks

The above is obviously a very simplistic model but, as I argued above, this has its advantages. By proposing a single cause for the observed phenomenon such a cause could be excluded as a major factor if the results were a long way off the observed effect. As it turns out, it appears that the proposition that the increase in CO_2 concentration in the atmosphere is largely responsible for the rise in mean global temperature cannot be so easily dismissed.

Of course, this conclusion depends on the reliability of the data, which I took from the NASA website [17], but one has to assume that these have been determined to the best scientific standards. It has to be realised, however, that this is no easy matter. The temperature across the globe obviously varies greatly, much more than the reported increase of *ca.* 0.7 ^{0}C over the period 1880-2008. Apparently, the values are obtained as averages over several thousand weather stations. A careful statistical analysis has to be made of the influence of individual stations. Moreover, the stations are obviously not distributed uniformly across the globe, so some weighted average has to be taken, and it is not immediately obvious what the right weighting should be. In fact, carbon dioxide in the atmosphere is very uniformly distributed over the Earth [19], so one would expect that the trend should be visible in the data from individual weather stations, albeit with higher fluctuations. That errors can easily be made in the statistical analysis of the data is illustrated by the 'hockey stick affair': see [7] and also [14]. An analysis of temperature proxy data going back to the year 1400 and even beyond was made by Mann et al. [11, 12], and appeared to demonstrate that the temperature had shown only moderate variations during the first half of the millennium. It featured strongly in one of the IPCC reports, but the statistical analysis was later shown to be faulty [15, 16]. (A more recent study [5] seems to uphold Mann et al.'s conclusions to some extent, however.)

Another complication is the uncertain influence of cloud formation. This could in fact be both positive, due to higher absorption, as well as negative, due to higher reflection. And there is also the question of whether higher CO_2 levels will increase vegetation thus reducing the increase in CO_2 concentration in the atmosphere. These factors are so uncertain that it is quite pointless including them in a model in some arbitrary way. All in all, based on the above model, I nevertheless tentatively conclude that, despite all the inherent uncertainties, it is more likely than not that most of the increase of the world mean temperature can be attributed to the rise in CO_2.

Apart from the physics, the main controversy is perhaps more about the necessity of drastic action to prevent further significant increase in CO_2 levels. In this regard, I think that Lord Lawson [10] makes a number of valuable practical observations, even if he is rather too dismissive of the science. First of all, it is highly unlikely that an agreement can be made where all major players, including India and China, commit to making significant cuts in their CO_2 emissions. The rapid economic development of these two countries in particular, with their large populations, makes an exponential rather than linear growth in CO_2 levels much more likely. Notice that, according to the simple model, the exponential scenario leads to double the rise in temperature in 2100 compared with a linear growth. Without the collaboration of these countries, the exercise is therefore rather futile. At the same time, drastic measures would do great damage to the economy, and with the current already difficult economic conditions, this could well be far more serious for future generations than global warming.

It is often suggested that global warming could lead to catastrophic disasters, which indeed justify drastic measures. (This was in particular emphasised in the highly sensationalist film by Mr. Al Gore.) So far there is very little sign of this happening on a 100-year time scale. For example, sea level rise is reported to be about 2 mm per year [6, 1]. (*A propos*, it puzzles me how this can be measured with any accuracy, given the large waves on the oceans and the varying tides. Apparently, these all average out sufficiently over the span of a year or so to make a reliable estimate of the trend. A good statistical analysis of the likely error is clearly necessary.) In any case, this is not dramatic at the present time. The increase due to expansion was estimated above, and is no cause for alarm.

A far more dramatic increase would of course result from a complete or partial collapse of the ice cover on Greenland or Antarctica. So far, there are no indications that this is likely any time soon. An increase in the movement of the glaciers on Greenland has been reported, but on the other hand the ice in the central higher regions seems to be stable [9]. A recent estimate of Greenland ice mass loss was made by Velicogna & Wahr [20], amounting to approx. 0.5 mm sea level rise per year. Even if this rate increases tenfold, then Greenland's contribution to sea level rise is still less than 25 cm in 100 years. If such a scenario would transpire it would lead to flooding in places, but the time span should nevertheless be long enough to build coastal defenses where needed. Similarly, on Antarctica there has been a very spectacular collapse of the Larsen and Wilkins ice shelves, but this does not mean that the rest of this immense continent is also unstable. In fact, some increase in ice thickness has been reported [21, 3]. Nevertheless, a net decrease in ice volume has also been measured [18]. All this seems to indicate that changes are indeed taking place but are only significant on a time-scale of centuries. Any consequences of such changes are better dealt with by adaptation than unrealistically large reductions in emissions. Of course, the physics of glaciers is still poorly understood, and a sudden instability in the form of an avalanche cannot be excluded. Such a low-probability but high risk possibility should be treated in the same way as other such disaster scenarios, like the impact of an asteroid: possible technological solutions for averting the danger should be explored without endangering the normal functioning of society.

Whereas trying to reduce emissions to any significant extent seems to be both impossible to achieve politically and prohibitively expensive economically, other measures *are* appropriate. Clearly, further research, in particular close monitoring of sea levels, temperatures, ice conditions on Greenland and

Antarctica, etc. is essential. Though I am sceptical about large-scale computer modelling, that does not mean that it should not be done. First of all, the programs are similar to those used for weather forecasting, which certainly is useful, and secondly, it is not costly and may yield spin-off benefits. The reliability of the resulting climate predictions is doubtful, however. More research into other sources of energy, and transporting and storing energy efficiently, as well as ways of capturing CO_2 should also be pursued. Building sea and river defenses would be a good preventative measure, esp. in places that are already at risk of flooding. Whereas investment in diversification of power generation is clearly sensible, various gimmicky measures like energy efficient light bulbs, banning SUVs and restricting flights are ineffective and cause more disruption than they are worth.

Finally, let me say that I have done this little piece of research purely out of personal interest. I am not supported by industry and have no relevant business interests. This is indeed the great advantage of being employed by an independent research institute with complete freedom to do research without external influences or the obligation to obtain grants. I am grateful to the Dublin Institute for Advanced Studies for this opportunity.

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