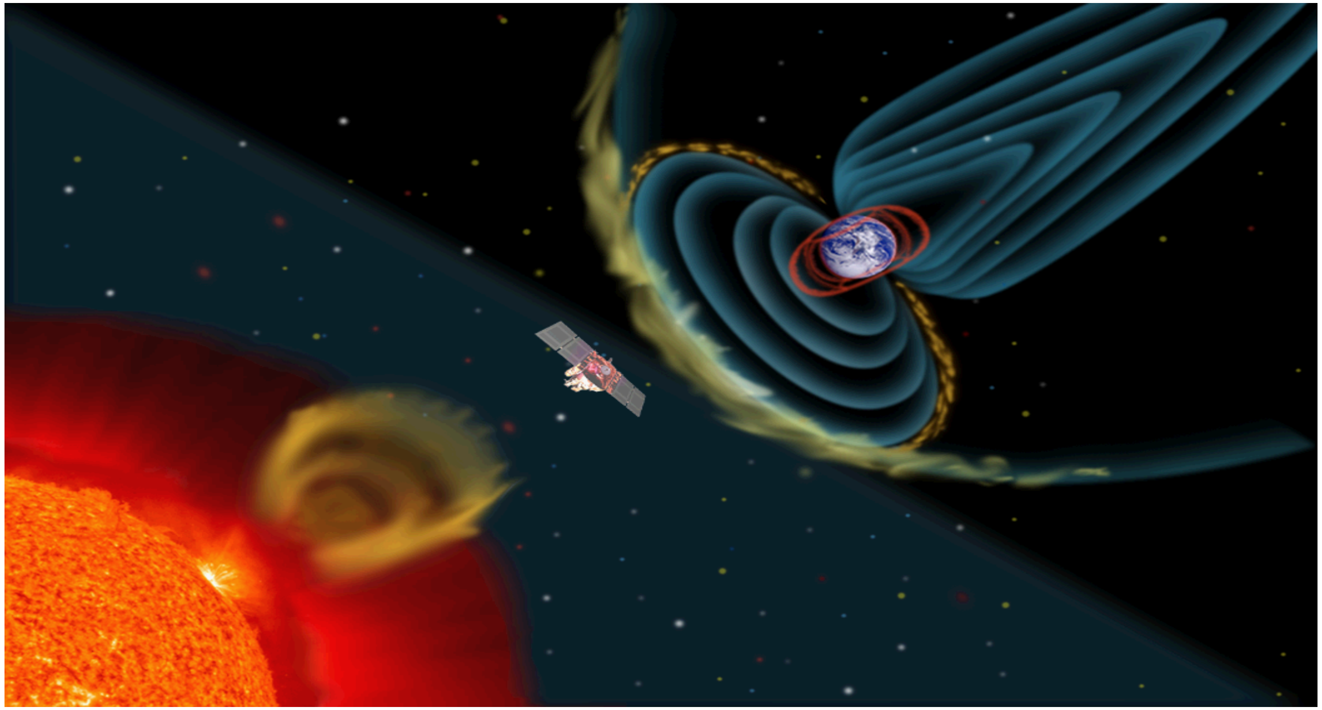
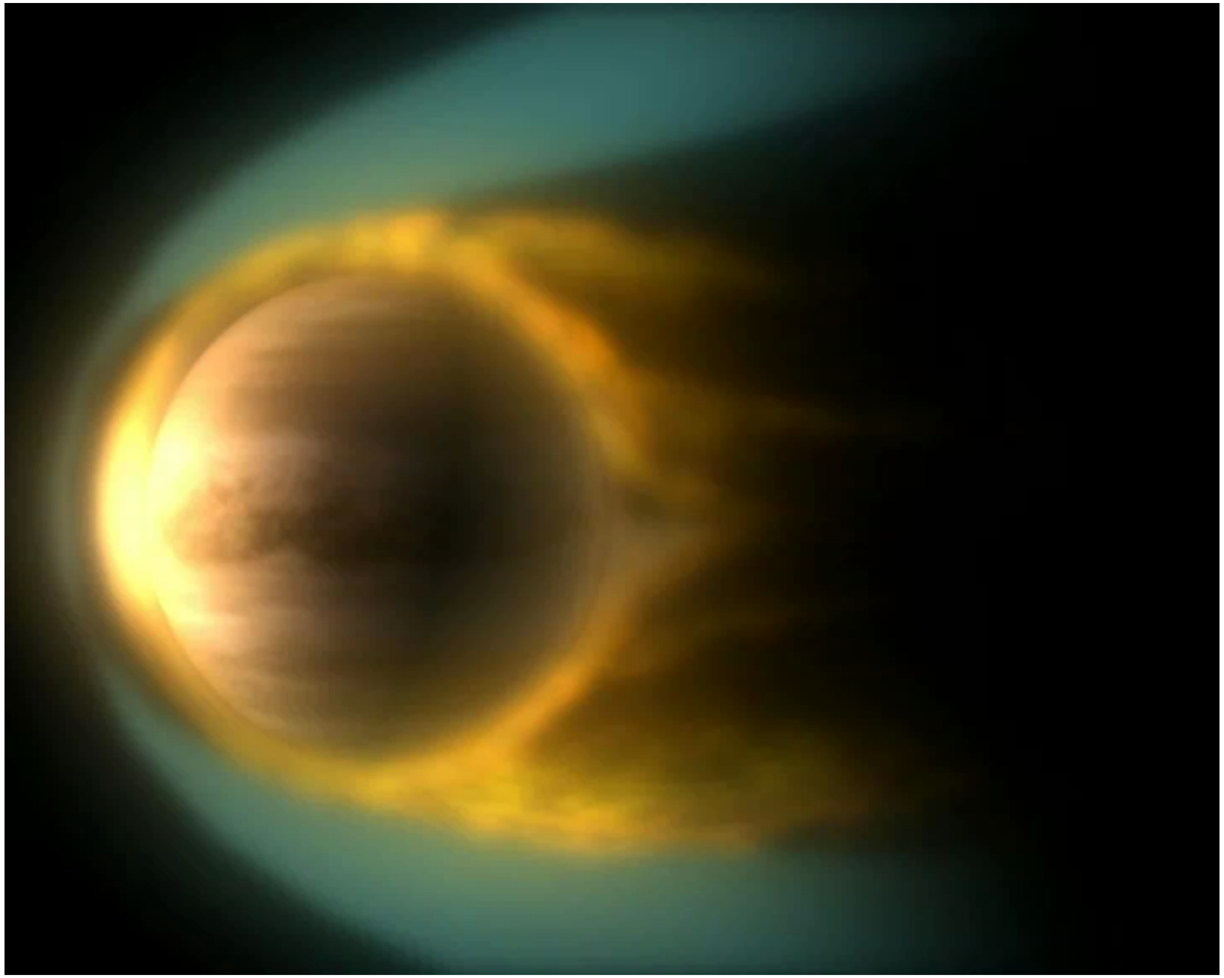
Lecture 14-15: Planetary magnetospheres



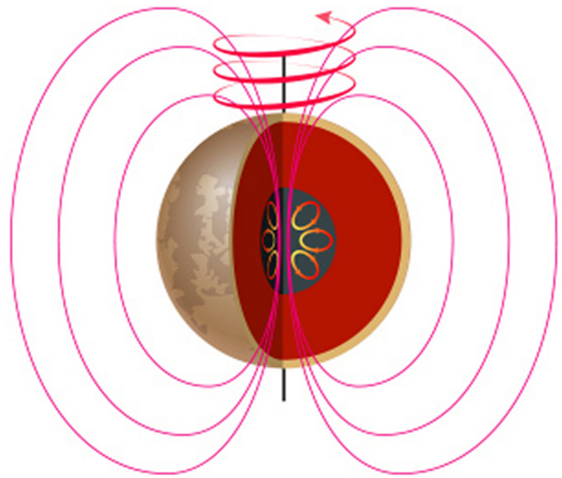
- Today's topics:
  - Planetary magnetic fields.
  - Interaction of solar wind with solar system objects.
  - Planetary magnetospheres.

Venus in the solar wind



- Click on "About Venus" at http://www.esa.int/SPECIALS/Venus_Express/

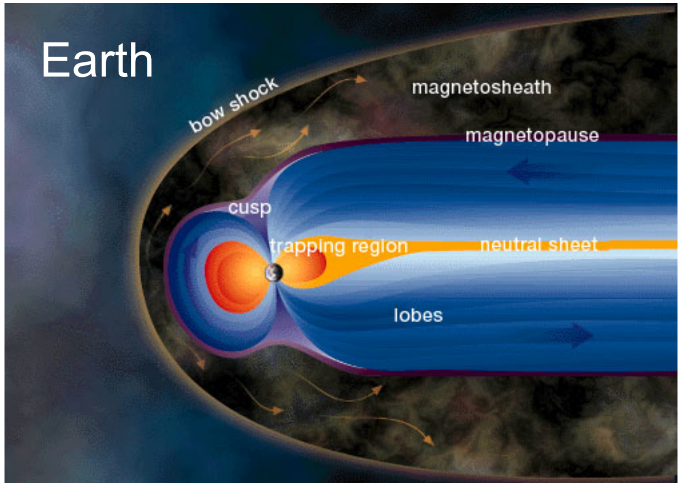
Planetary magnetism

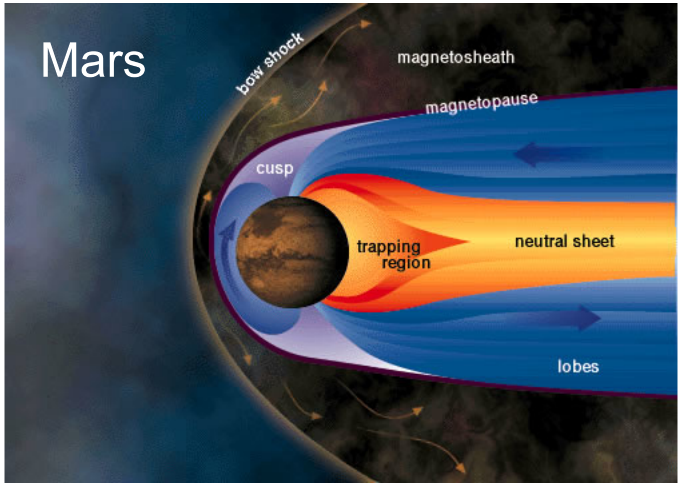


- Conducting fluid in motion generates magnetic field.
- Earth's liquid outer core is conducting fluid => free electrons are released from metals (Fe & Ni) by friction and heat.
- Variations in the global magnetic field represent changes in fluid flow in the core.
- Defined magnetic field implies a planet has:
  1. A large, liquid core
  2. A core rich in metals
  3. A high rotation rate
- These three properties are required for a planet to generate an *intrinsic magnetosphere*.

Planetary magnetism

- *Earth:* Satisfies all three. Earth is only terrestrial planet with a strong *B*-field.
- *Moon:* No *B*-field today. It has no core or it solidified and ceased convection.
- *Mars:* No *B*-field today. Core solidified.
- *Venus:* Molten layer, but has a slow, 243 day rotation period => too slow to generate field.
- *Mercury:* Rotation period 59 days, small *B*-field. Possibly due to large core, or magnetised crust, or loss of crust on impact.
- *Jupiter:* Has large *B*-field, due to large liquid, metallic core, which is rotating quickly.



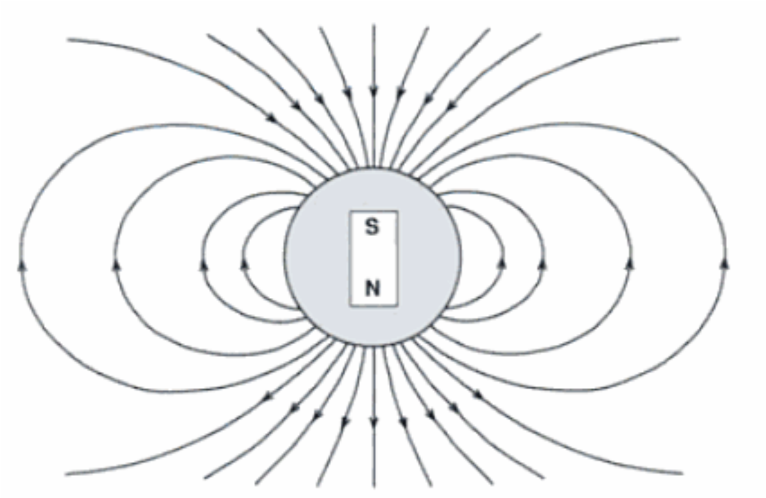


Planetary magnetic fields

- Gauss showed that the magnetic field of the Earth could be described by:

$$\hat{B} = -\mu_0 \hat{\nabla} V(\hat{r})$$
$$= -\mu_0 \hat{\nabla}(V^i + V^e)$$

where V^i is the magnetic scalar potential due to sources inside the Earth, and V^e is the scalar potential due to external sources.



- For a pure dipole field,

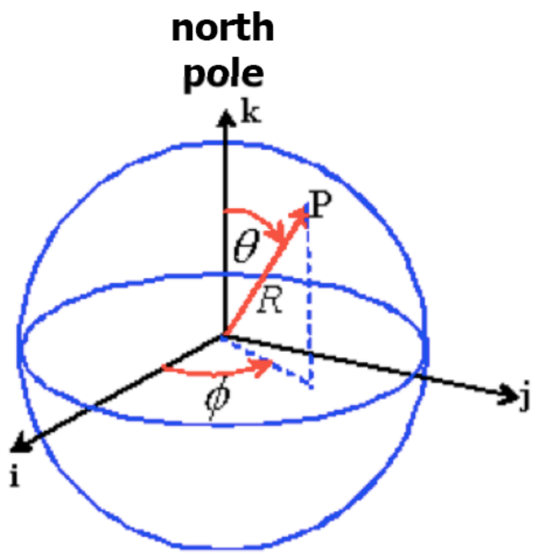
$$V(r) = \frac{1}{4\pi r^3} \hat{M} \cdot \hat{r} = \frac{M \cos\theta}{4\pi r^2}$$

where M is the planetary dipole moment.

- For Earth, M = 8 x 10^{15} T m^3 or 30.4 ? T R_E^3

Planetary magnetic fields

- In spherical polar coordinates,



$$R = (x^2 + y^2 + z^2)^{1/2} \quad \text{(radius)}$$

$$\boxed{?} = \cos^{-1}(z/R) \quad \text{(colatitude; 0 - } \boxed{?}\text{)}$$

$$\boxed{?} = \tan^{-1}(y/x) \quad \text{(longitude; 0 - 2}\boxed{?}\text{)}$$

where $x = R \sin\boxed{?} \cos \boxed{?}$

$y = R \sin\boxed{?} \sin \boxed{?}$

$z = R \cos\boxed{?}$

- The gradient operator in spherical polar coordinates is

$$\nabla f = \left(\frac{df}{dr}, \frac{1}{r}\frac{df}{d\theta}, \frac{1}{r\sin\theta}\frac{df}{d\phi} \right)$$

Planetary magnetic fields

- The magnetic field can therefore be calculated using

$$\hat{B} = -\mu_0 \hat{\nabla} V(\hat{r})$$
$$= -\mu_0 \left(\frac{dV}{dr}, \frac{1}{r}\frac{dV}{d\theta}, \frac{1}{r\sin\theta}\frac{dV}{d\phi} \right)$$

- Therefore the three component of B (= (B_r, $B_{\boxed{?}}$, $B_{\boxed{?}}$)) are:

$$B_r = -\frac{2\mu_0 M \cos\theta}{4\pi r^3}$$
$$B_\theta = -\frac{\mu_0 M \sin\theta}{4\pi r^3}$$
$$B_\phi = 0$$

- The total field is

$$B(r,\theta,\phi) = \sqrt{B_r^2 + B_\theta^2 + B_\phi^2}$$
$$= \frac{\mu_0 M}{4\pi r^3}\sqrt{1 + 3\cos^2\theta}$$

Planetary magnetic fields

- At the north pole, $B_r(r,0,\phi) = -\dfrac{\mu_0 M}{2\pi r^3}$ and $B_\theta(r,0,\phi) = 0$
- At the equator, $B_r(r,90,\phi) = 0$ and $B_\theta(r,90,\phi) = -\dfrac{\mu_0 M}{4\pi r^3}$
- At the Earth's surface, $r = R_E$ => $B_r = 2B_0 \cos \boxed{?}$

  $B_{\boxed{?}} = B_0 \sin \boxed{?}$

  where $B_0 = -\dfrac{\mu_0 M}{4\pi R_E^3}$
- B_0 is the equatorial strength of field at surface.
- Typically on Earth, B_0 =-30.7 μT or -0.307 Gauss.

Planetary magnetic fields

Table 5.2 *Magnetic characteristics of the planets (data source: Van Allen and Bagenal, 1999)*

Planet	Mean orbital radius [AU]	Mean radius of planet [km]	Period of rotation [days]	Magnetic dipole moment [m_E]	Equivalent equatorial magnetic field [nT]	Dipole tilt to rotation axis [°]
Mercury	0.3830	2,440	58.81	0.0007	300	14
Venus	0.7234	6,052	243.7(R)	< 0.0004	< 3	—
Earth	1	6,371	1	1	30,500	10.8
Moon	0.00257	1,738	27.32	—	—	—
Mars	1.520	3,390	1.0275	< 0.0002	< 30	—
Jupiter	5.202	69,910	0.414	20,000	428,000	9.6
Saturn	9.576	58,230	0.444	600	22,000	< 1
Uranus	19.19	25,362	0.720(R)	50	23,000	58.6
Neptune	30.05	24,625	0.671	25	14,000	47

- Source: *Fundamentals of Geophysics* (Lowrie)

Solar wind effects

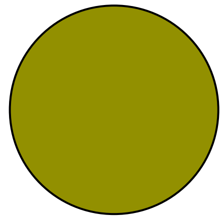
- Depends on *magnetic field* and *atmospheric* properties of object.

- Planets with magnetic fields essentially have a dipole field ($B(r) \sim 1 / r^3$).

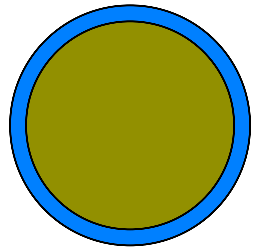
- Consider two types of magnetosphere:

  1. Induced magnetosphere - solar wind interaction creates a magnetosphere.

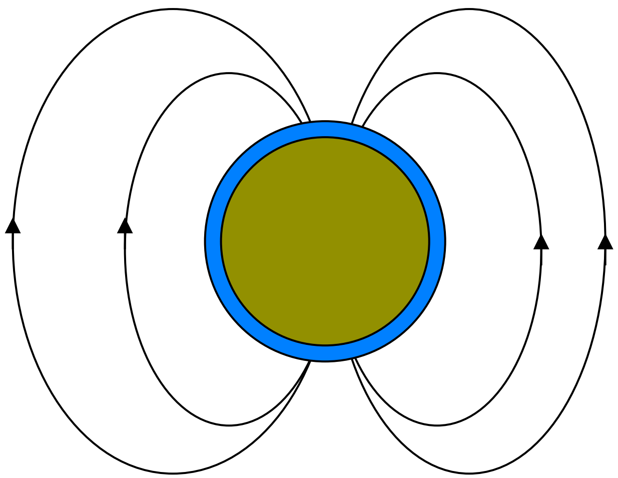
  2. Intrinsic magnetosphere - object generates its own magnetic field.



No magnetic field or atmosphere



No magnetic field but atmosphere



Magnetic field and atmosphere

Pressure due to the solar wind

- Solar wind exerts *magnetic* and *dynamic* pressure on objects (comets, planets, etc.) in the solar system.
  - Magnetic pressure: $P_B = B^2 / 2\boxed{?}_0$
  - Dynamic pressure: $P_D = 1/2\ \boxed{?}\ v^2$

- Sun's field is a dipole: $B = B_S / r^3$, where B_S is the dipole moment at the equator.

  $$=> P_B = B_S^2 / 2\boxed{?}_0 r^6$$

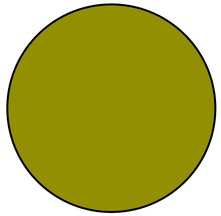
- The solar wind density $\sim r^{-2}$.

- As $P_B \sim r^{-6}$ and $P_D \sim r^{--2}$ => $P_D >> P_B$ at large distances from the Sun.

  => only consider dynamic pressure of solar wind on objects.

Induced magnetospheres

- No field, no atmosphere
  - Solar particles encounter surface of body and are absorbed or bounce back.
  - Can lead to evaporation and outgassing of material (e.g., comet nucleus and coma).
  - Pressure due to outgassing reaches balance with solar wind: $1/2\ \boxed{?}_g\, v_g^{\,2} = 1/2\ \boxed{?}_{sw}\, v_{sw}^{\,2}$

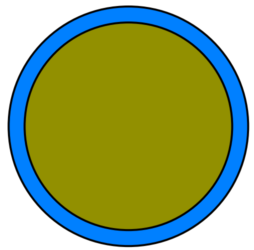


No magnetic field or atmosphere

- No field, but atmosphere present
  - Atmosphere has gas pressure which balances the solar wind pressure:

$$P_{pa} = P_{sw}$$

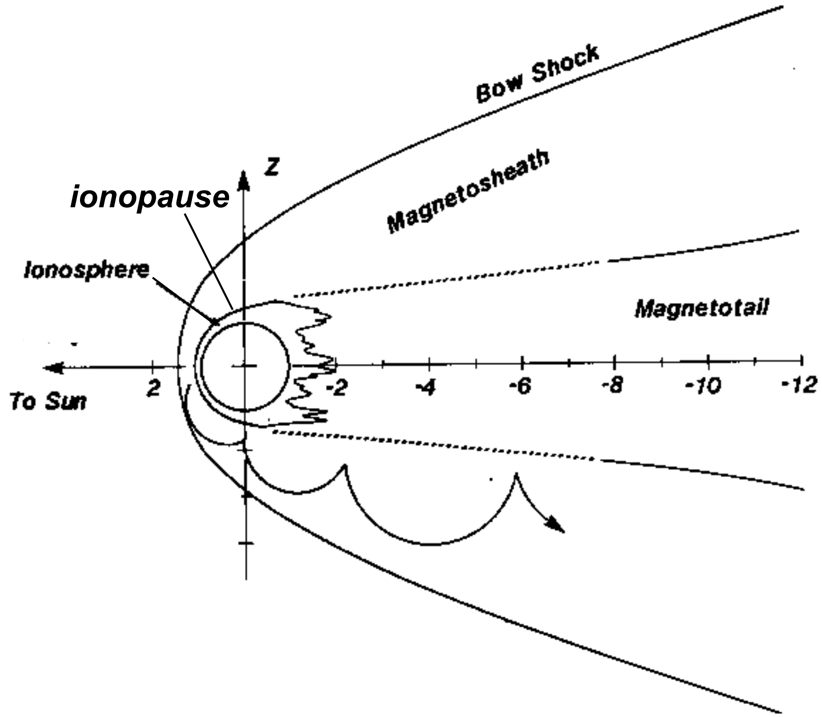
  - This occurs at *ionopause.*
  - Occurs on Venus and Mars, neither of which have significant intrinsic magnetic fields.



No magnetic field but atmosphere

Induced magnetospheres (cont.)

- Solar EUV radiation ionizes upper atmospheres of planets.
- If thermal pressure of ionosphere equals solar wind dynamic pressure, then ionosphere can balance the solar wind pressure.
- *Magnetosheath* forms above the *ionosphere* and deflects the solar wind.
- *Ionopause* separates the ionosphere from the magnetosheath.
- Solar wind is supersonic and thus forms a detached *bow shock*.



What is the height of the ionopause?

- Assuming hydrostatic equilibrium in the planetary atmosphere:

$$\frac{dP}{dr} = -\rho g$$

- As $\boxed{?}$ = $n\ m$ and $P = n\ k\ T$ => $\boxed{?}$ = $P\ /\ k\ T$, and we can write:

$$\frac{dP}{dr} = -\frac{Pmg}{kT}$$

- Rearranging and integrating:

$$\int_{P_0}^{P} \frac{dP}{P} = -\frac{mg}{kT}\int_{r_0}^{r} dr$$

r_0 = radius of planet
P_0 = pressure at surface

$$\ln\left(\frac{P}{P_0}\right) = -\frac{mg}{kT}(r - r_0)$$

$$=> P = P_0 e^{-\frac{mg}{kT}(r-r_0)}$$

- Letting, $H = kT\ /\ mg$ and $P = P_{pa}$ (H is the scale height): $\boxed{P_{pa} = P_0 e^{-(r-r_0)/H}}$

- This is the pressure as a function of height from surface of a planet's atmosphere.

What is the height of the ionopause? (cont.)

- The ionopause occurs at a height where the pressure due to the solar wind equals the pressure of the planetary atmosphere, i.e., $P_{sw} = P_{pa}$

- The dynamic pressure due to the solar wind is $P_{sw} = 1/2\ \boxed{?}_{sw}\ v_{sw}^{2} = 1/2\ n_{nm}\ m_{sw}\ v_{sw}^{2}$.

- Therefore,

$$\frac{1}{2} n_{sw} m_{sw} v_{sw}^{2} = P_0 e^{-(r-r_0)/H}$$

$$\boxed{\therefore r - r_0 = H \ln\left(\frac{2P_0}{n_{sw} m_{sw} v^{2}{}_{sw}}\right)}$$

- At Mars, $n_{sw} = 1 \times 10^{6}\ m^{-3}$, $v_{sw} = 330\ km\ s^{-1}$, $T = 200\ K$ (planet surface temperature), $n_{pa} = 3 \times 10^{10}\ m^{-3}$, $r_0 = 3393\ km$. What is r-r_0?

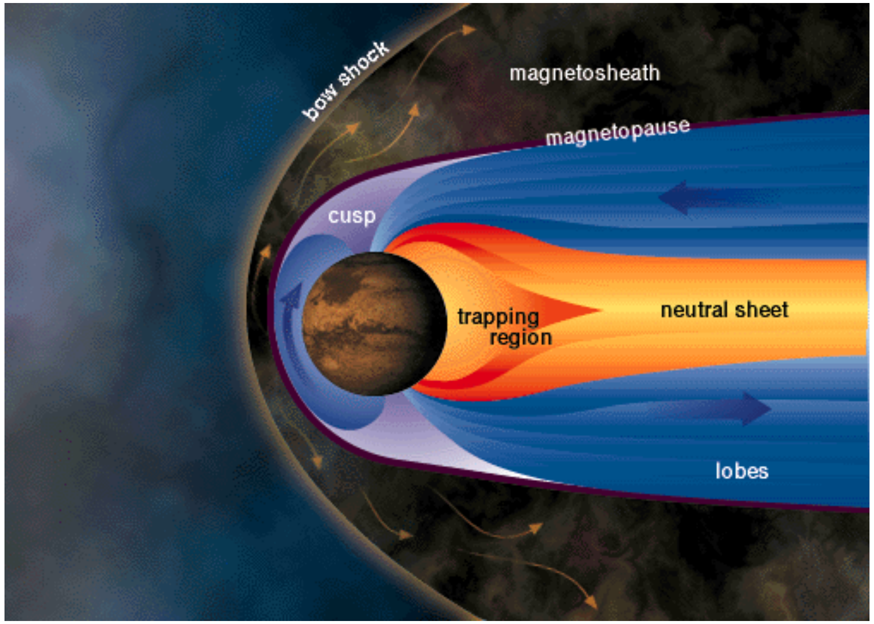
- On Mars, r is so small that solar wind particles reach the surface. What are the implications for humans on Mars?

Magnetospheres of mars

- Martian atmosphere diverts the solar wind, because Mars lacks a significant planetary magnetic field (it's internal dynamo shut off).

- Mars is an "*induced obstacle*"; the ionosphere interacts with the solar wind.

- Very unlike Earth, which is encapsulated within an *intrinsic magnetosphere*. This magnetosphere buffers us from charged particles in the solar wind.



Mars in the solar wind

Induced magnetospheres of planets

- ***Venus:*** The magnetic moment is less than one hundred thousandths that of Earth. Plays no role in the solar wind interaction with the planet. Still do not know how much atmosphere is being lost to the solar wind.

- ***Mars:*** Precise size of the magnetic field is not known but its strength is much less than one ten thousandths of Earth. Like Venus, the intrinsic magnetic field is not significant for the solar wind interaction. The ionosphere is thought to be magnetized because the solar wind dynamic pressure exceeds the thermal pressure of the ionosphere. Other features, such as the bow shock and magnetotail, are very similar to those of Venus.

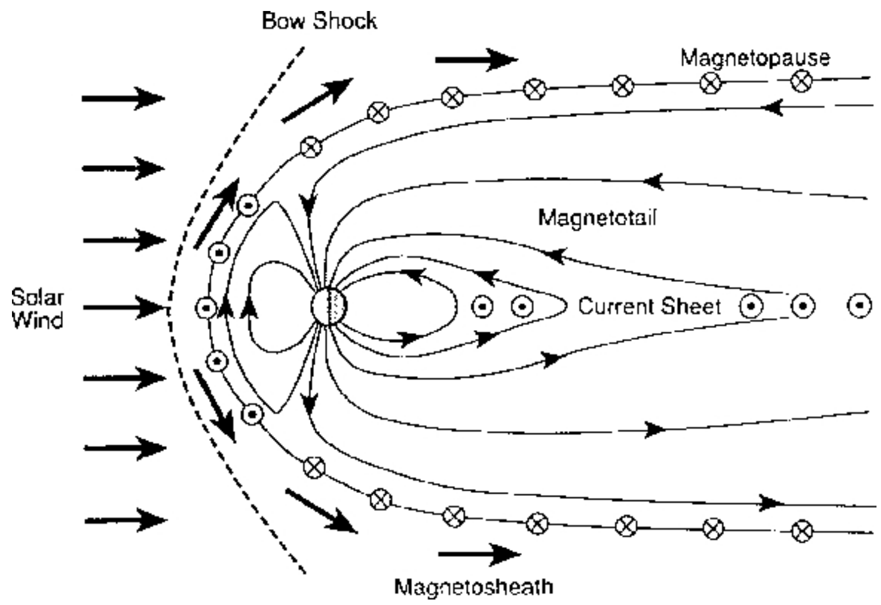
- ***Comets:*** Comets are much smaller objects than planets if only their nuclei are considered. The size over which the cometary gas can spread in the solar wind is thus controlled by the speed of expansion of the cometary gas (about one km/s) and the ionization time (about a day at 1 AU from the Sun).

Intrinsic magnetospheres

- Geomagnetic field of many planets can be approximated by a dipole. The forcing by the solar wind modifies this field, creating a cavity called the *magnetosphere*.

- Magnetosphere shelters surface from high energy solar wind.

- Outer boundary of magnetosphere is called the *magnetopause*.



- In front of dayside magnetopause another boundary called the *bow shock* is formed because solar wind is supersonic. Region between bow shock and magnetopause is *magnetosheath*.

What is the height of the magnetopause?

- Magnetopause located where planet's magnetic field pressure balances pressure due to the solar wind: $P_B = P_{sw}$.

- Magnetic pressure is $P_B = B^2 / 2\boxed{?}_0$

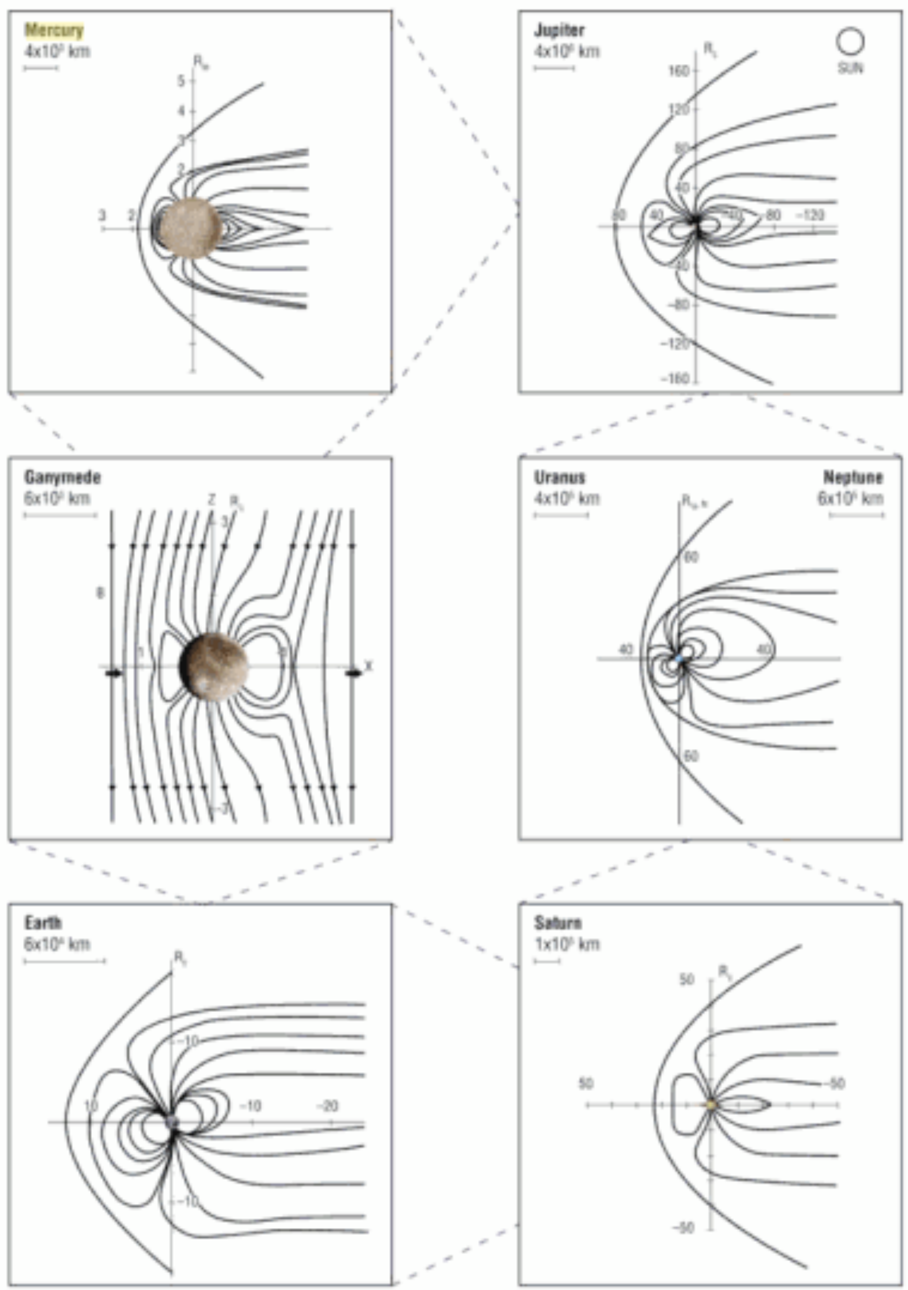
- At equator, magnetic field varies as $B(r) = B_0\,(R_p/r)^3$, where B_0 is the surface field strength. Therefore,

$$P_B = B_0^2 / 2\boxed{?}_0\,(R_p/r)^6$$

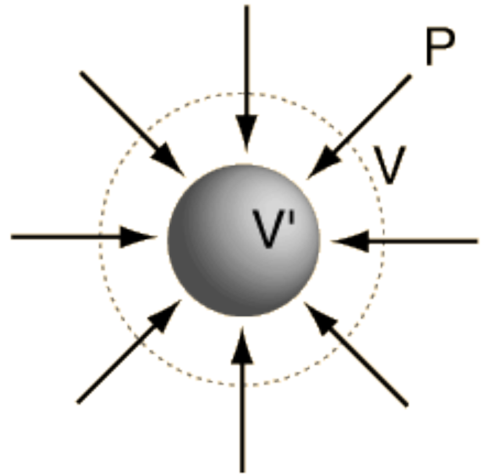
- Using, $P_{sw} = 1/2\ \boxed{?}_{sw}\, v_{sw}^2 \Rightarrow$ $\qquad 1/2\ n_{sw}\, m_p\, v_{sw}^2 = B_0^2 / 2\boxed{?}_0\,(R_p/r)^6$

- Stand-off height of magnetopause therefore:

$$\boxed{r_{so} = \left(\frac{B_0^2}{\mu_0 n_{sw} m_p v_{sw}^2} \right)^{1/6} R_p}$$


Mercury
4x10³ km
Jupiter
4x10⁶ km
SUN
Ganymede
6x10³ km
Uranus
4x10⁵ km
Neptune
6x10⁵ km
Earth
6x10⁴ km
Saturn
1x10⁵ km

Magnetosphere compressibility



- Compressibility of magnetosphere can be understood by considering the compressibility:

$$\kappa = \frac{1}{V}\frac{dV}{dP}$$

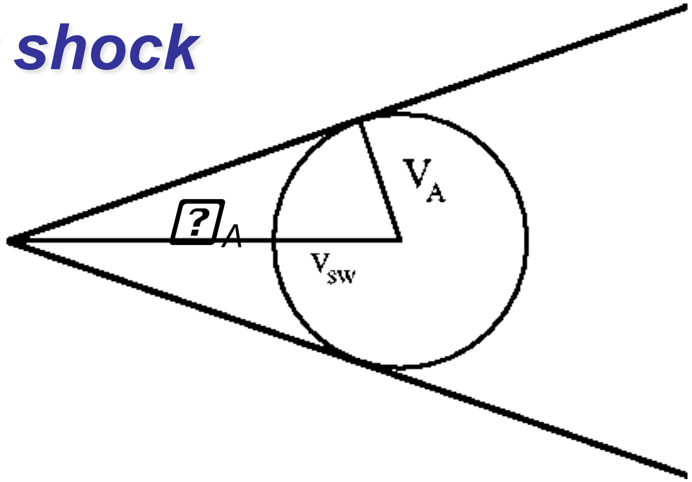
  where V is the volume of the magnetosphere, and P is the external pressure.

- As $V_{mp} = 4/3\ \boxed{?}\ r_{mp}{}^{3} \Rightarrow \kappa = \frac{1}{r_{mp}}\frac{dr_{mp}}{dP}$

$$\Rightarrow \kappa \propto \frac{1}{P_{ram}}$$

- Mercury has a very "stiff" magnetosphere, whereas Jupiter does not (see "Mercury" by Balogh et al. on Google Books).

Shape of the bow shock



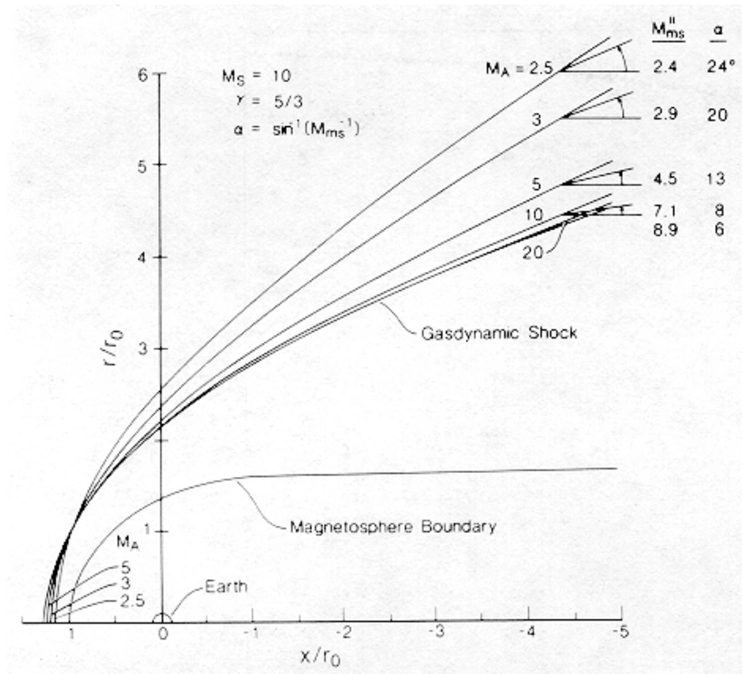
- Solar wind is both supersonic and super-Alfvenic at large distances from Sun. i.e., $v_{sw} >> v_s$ and $>> v_A$ ($= B/\sqrt{\mu_0 \rho}$)

- In fact, $M_A = v_{sw} / v_A \sim M_s = v_{sw} / v_s \sim 8$.

- Changes in shock shape can be understood using *Mach cone*:

$$\theta_A = \sin^{-1}\left(\frac{v_A}{v_{sw}}\right) = \sin^{-1}\left(M^{-1}\right)$$

- Thus, the shock shape becomes more blunt for smaller M_A and more swept back for larger M_A.



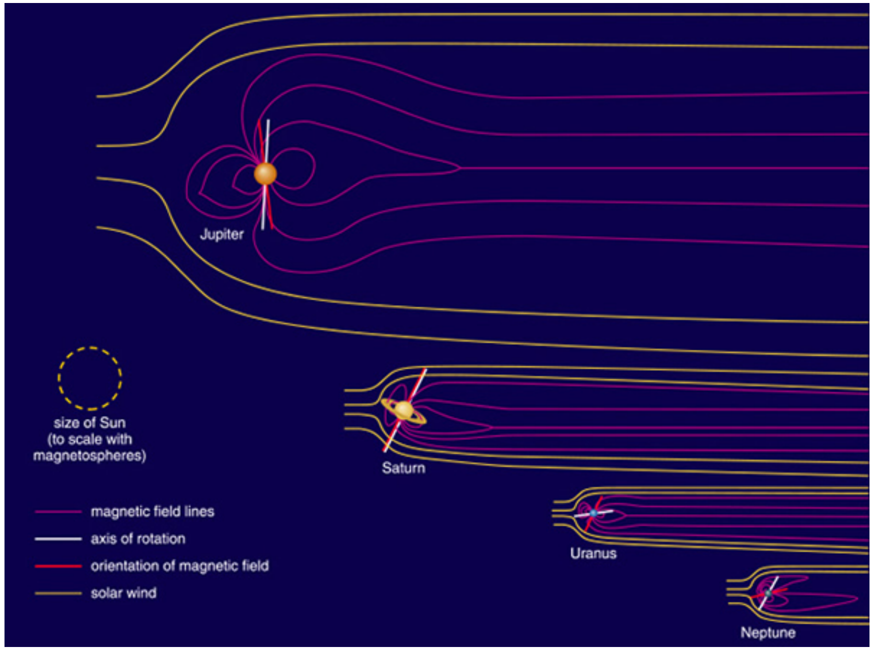
Intrinsic magnetospheres of the planets

- *Mercury:* Magnetic moment is ~1/3000th of Earth's. Equatorial surface magnetic field strength is ~250 *nT*.

- *Earth:* Surface field is ~31,000 *nT*.

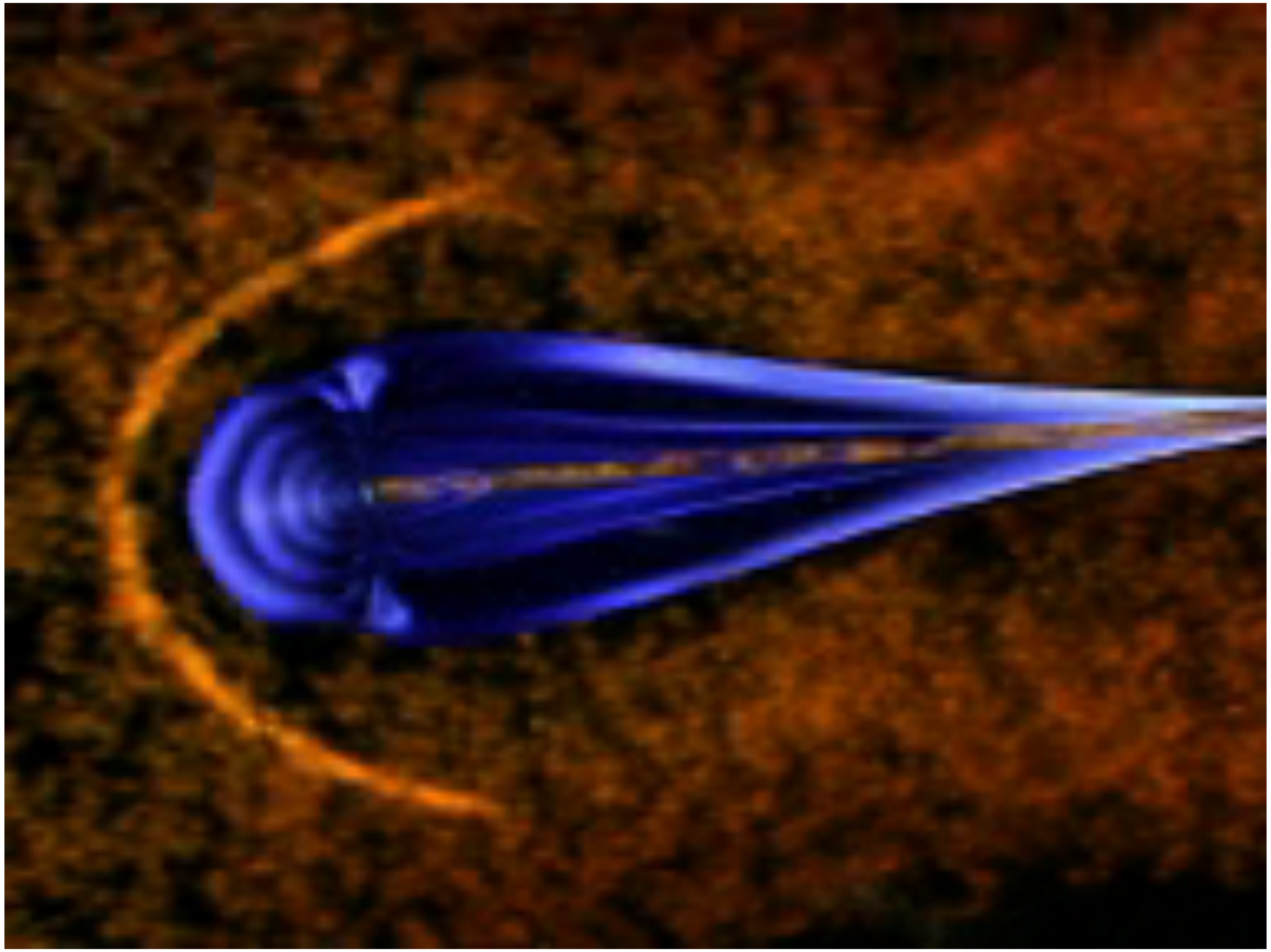
- *Jupiter:* Magnetic moment is largest of planets at ~10,000 times Earth's. Strength of field combined with weakness of wind at Jupiter produces enormous magnetosphere.

- *Saturn:* Since Saturn is smaller planet, its core in which the planetary magnetic field is generated is smaller => so is magnetic field. Magnetic moment of Saturn is 580 times that of Earth.

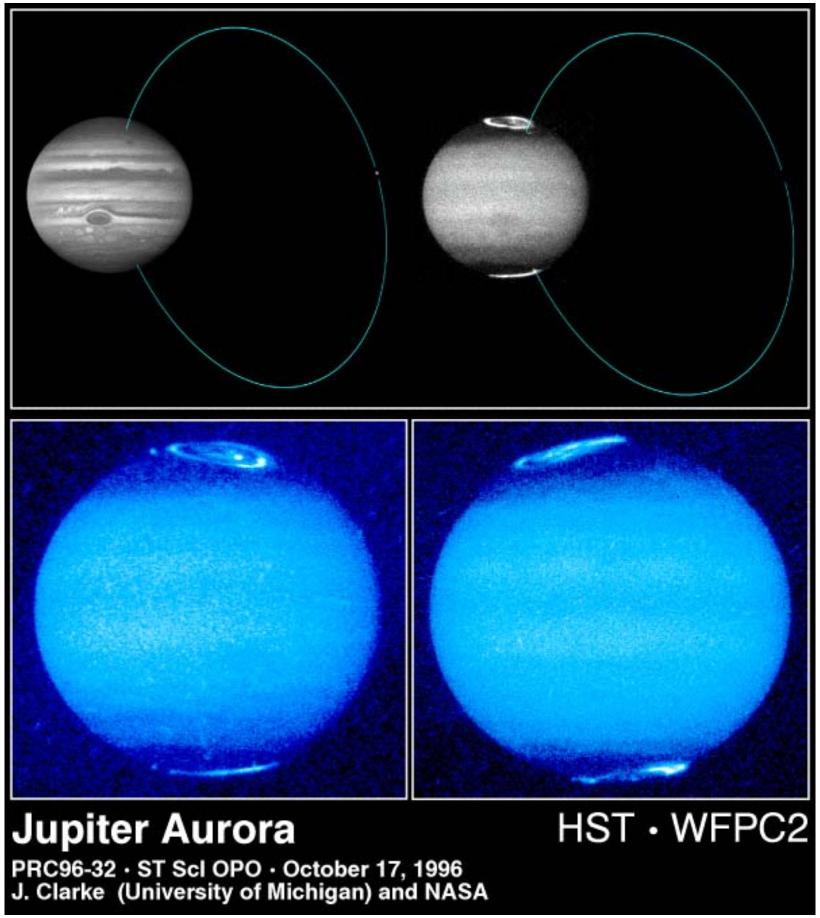


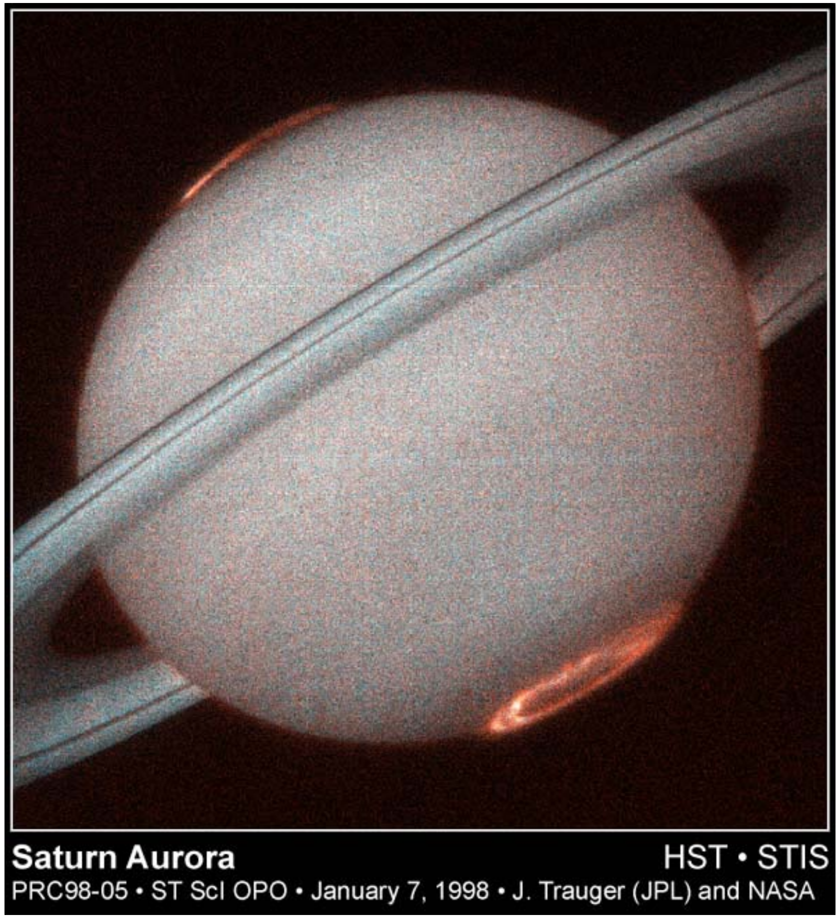
- *Uranus and Neptune:* Magnetic fields are irregular and not be well represented by a simple dipole. Magnetic moments are ~40 times < Earth's. Reason weakness and irregularity may be that the magnetic field is generated in salty ice/water oceans closer to the surface.

Earth's intrinsic magnetosphere



Aurorae of Jupiter and Saturn



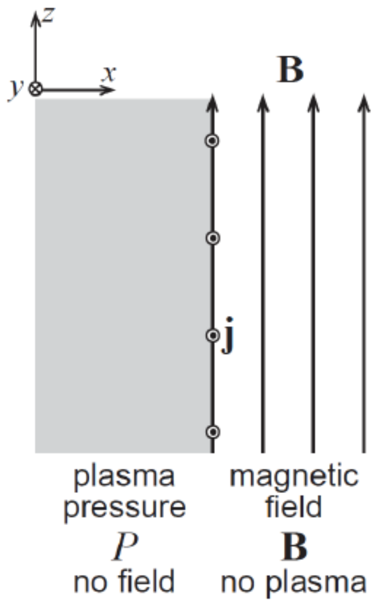


The Parker Spiral

	Mercury	Earth	Jupiter	Saturn
Equatorial radius/km	2440	6378	71323	60268
Equatorial surface magnetic field strength/nT	300	31000	426400	21100
Dipole offset from spin pole/deg	5 (?)	11	10	1
Orientation of equatorial magnetic field	South	South	North	North
Rotation period/h	1408	24	9.9	10.8 (?)
Distance from Sun/AU	0.35	1	5.2	9.5
Solar wind IMF/nT	20	7	1	0.7
Solar wind dynamic pressure/nPa	20	4	0.05	0.01
Solar wind magnetosonic Mach number	4	6	9	10
Stand-off distance				

Size of the Magnetospheric Cavity

The distance to the sub-solar magnetopause is determined by pressure balance between the solar wind flow and IMF on one side and the magnetospheric field and plasma on the other:



~ Solar wind vs Magnetosphere

Recall the (simplified) Eqtn of Motion for plasma:

$$\rho \frac{d\mathbf{v}}{dt} = \rho \mathbf{g} - \nabla P + \mathbf{j} \times \mathbf{B}$$

At the boundary we will assume equilibrium, negligible effect of gravity, and no field bending:

$$\nabla \left(P + \frac{B^2}{2\mu_0} \right) = 0$$

So, effectively dynamic and magnetic pressures balance on either side of the boundary:

$$P_1 + \frac{B_1^2}{2\mu_0} = P_2 + \frac{B_2^2}{2\mu_0}$$