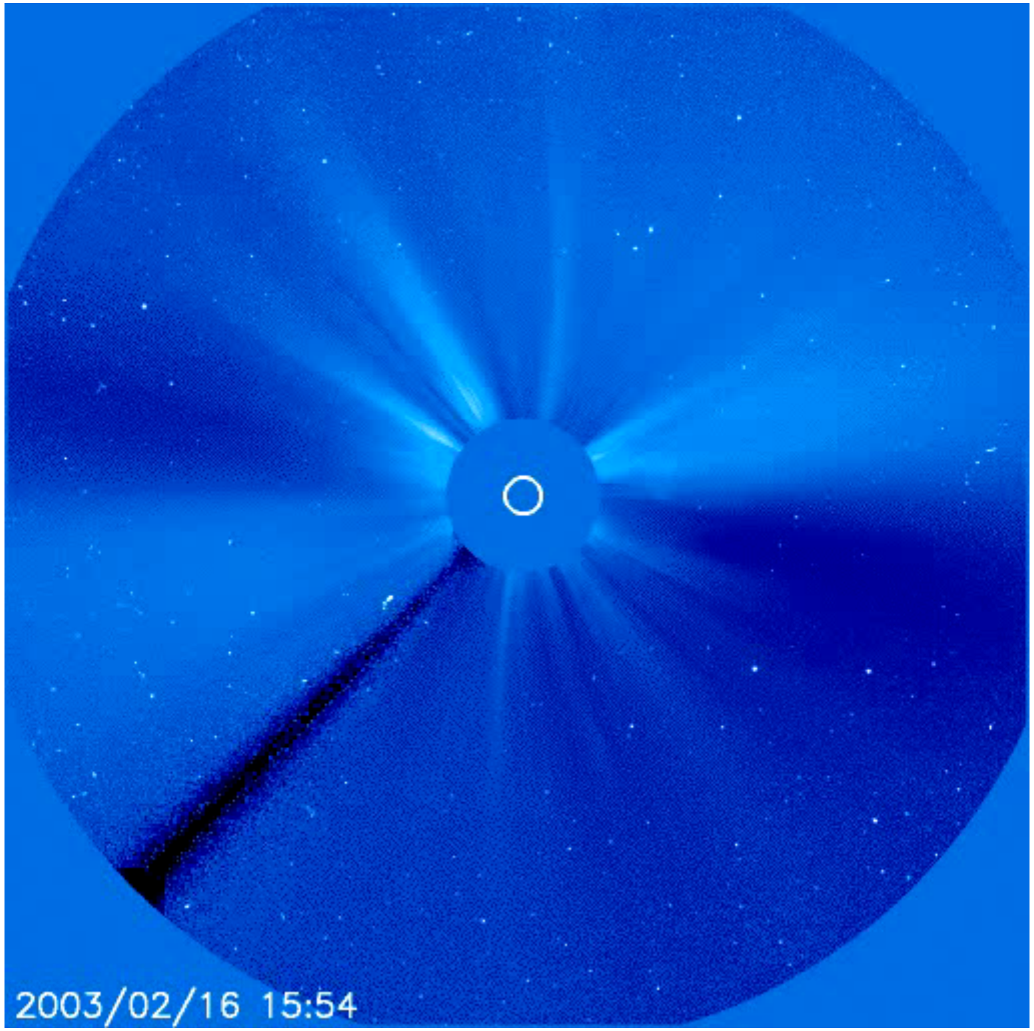
Lecture 17: The solar wind

- Topics to be covered:
  - Solar wind
  - Inteplanetary magnetic field



The solar wind

- Biermann (1951) noticed that many comets showed excess ionization and abrupt changes in the outflow of material in their tails - is this due to a solar wind?

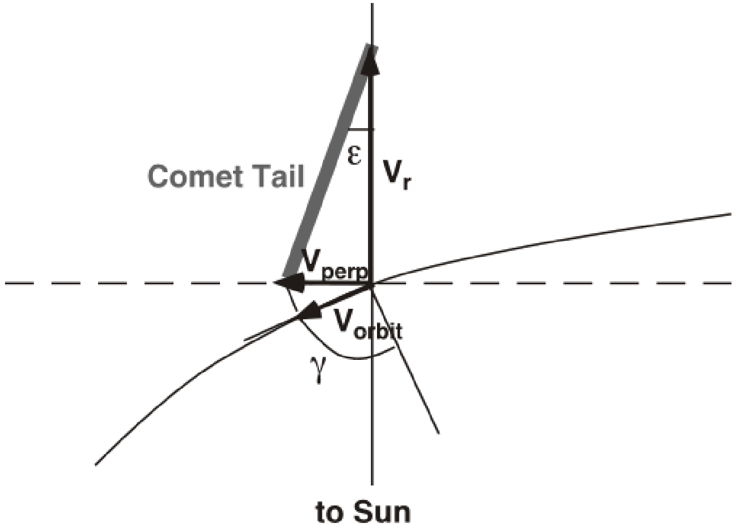
- Assumed comet orbit perpendicular to line-of-sight (v_{perp}) and tail at angle $\boxed{?}$ =>

$$\tan\boxed{?} = v_{perp}/v_r$$

- From observations, $\tan\boxed{?} \sim 0.074$

- But v_{perp} is a projection of v_{orbit}
  => $v_{perp} = v_{orbit} \sin\boxed{?} \sim 33\ km\ s^{-1}$

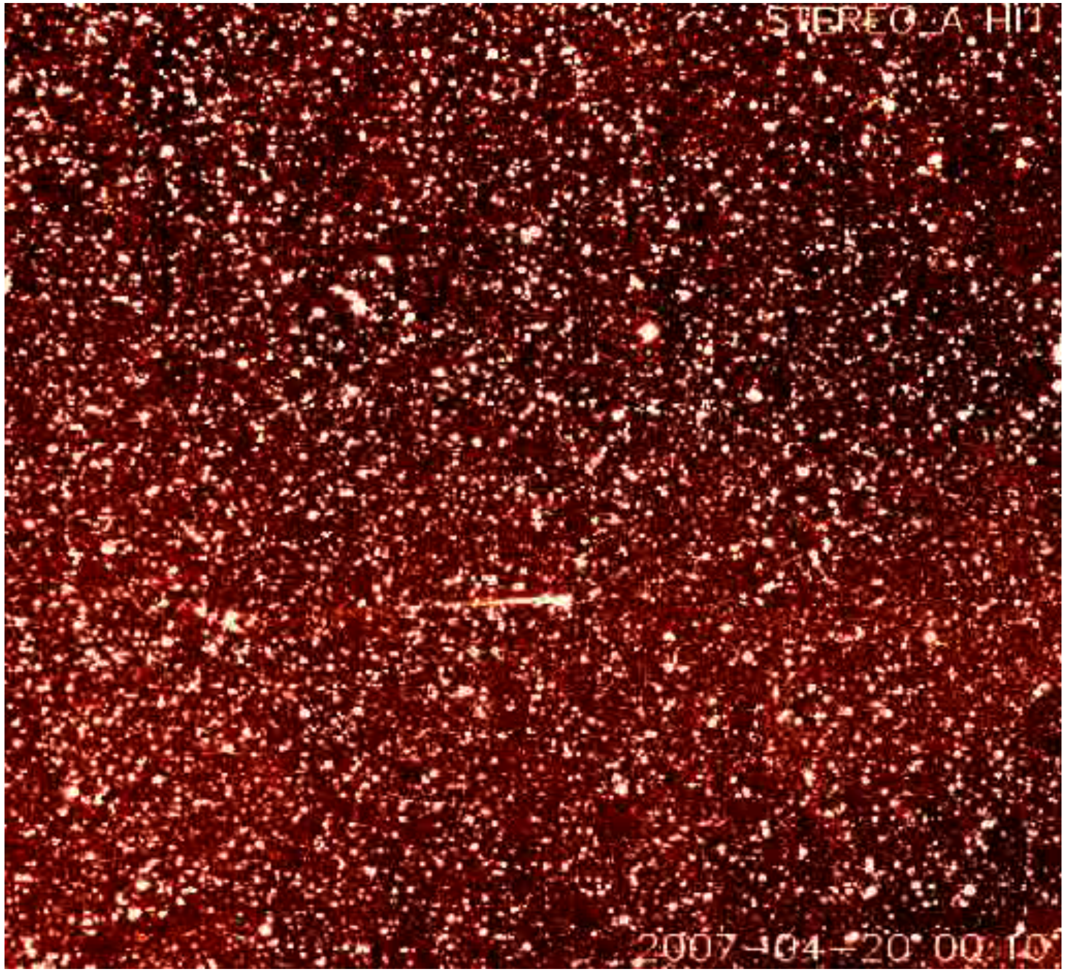
- From 600 comets, $v_r \sim 450\ km\ s^{-1}$.



- See Uni. New Hampshire course (Physics 954) for further details:
  http://www-ssg.sr.unh.edu/Physics954/Syllabus.html

The solar wind

- STEREO satellite image sequences of comet tail buffeting and disconnection.



Parker's solar wind

- Parker (1958) assumed that the outflow from the Sun is steady, spherically symmetric and isothermal.

- As $P_{Sun} >> P_{ISM}$ => must drive a flow.

- Chapman (1957) considered corona to be in hydrostatic equibrium:

$$\frac{dP}{dr} = -\rho g$$

$$\frac{dP}{dr} + \frac{GM_S\rho}{r^2} = 0 \qquad \textit{Eqn. 1}$$

- If first term >> than second => produces an outflow:

$$\frac{dP}{dr} + \frac{GM_S\rho}{r^2} + \rho\frac{dv}{dt} = 0 \qquad \textit{Eqn. 2}$$

- This is the equation for a steadily expanding solar/stellar wind.

Parker's solar wind (cont.)

- As, $\dfrac{dv}{dt} = \dfrac{dv}{dr}\dfrac{dr}{dt} = \dfrac{dv}{dr}v \quad => \dfrac{dP}{dr} + \dfrac{GM_S\rho}{r^2} + \rho v\dfrac{dv}{dr} = 0$

  or
$$\boxed{v\frac{dv}{dr} + \frac{1}{\rho}\frac{dP}{dr} + \frac{GM_S}{r^2} = 0} \qquad \textit{Eqn. 3}$$

- Called the *momentum equation*.

- *Eqn. 3* describes acceleration (1st term) of the gas due to a pressure gradient (2nd term) and gravity (3rd term). Need *Eqn. 3* in terms of *v*.

- Assuming a perfect gas, $P = R\,\boxed{?}\,T\,/\,\boxed{?}$ (*R* is gas constant; $\boxed{?}$ is mean atomic weight), the 2nd term of *Eqn. 3* is:

$$\frac{dP}{dr} = \frac{R\rho}{\mu}\frac{dT}{dr} + \frac{RT}{\mu}\frac{d\rho}{dr}$$

Isothermal wind => $dT/dr\ \boxed{?}\ 0$
$$\Rightarrow \frac{1}{\rho}\frac{dP}{dr} = \left(\frac{RT}{\mu}\right)\frac{1}{\rho}\frac{d\rho}{dr} \qquad \textit{Eqn. 4}$$

Parker's solar wind (cont.)

- Now, the mass loss rate is assumed to be constant, so the *Equation of Mass Conservation* is:

$$\frac{dM}{dt} = 4\pi r^2 \rho v = const \Rightarrow r^2 \rho v = const \qquad \textit{Eqn. 5}$$

- Differentiating,

$$\frac{d(r^2\rho v)}{dr} = r^2 \rho \frac{dv}{dr} + \rho v \frac{dr^2}{dr} + r^2 v \frac{d\rho}{dr} = 0$$

$$=> \frac{1}{\rho}\frac{d\rho}{dr} = -\frac{1}{v}\frac{dv}{dr} - \frac{2}{r} \qquad \textit{Eqn. 6}$$

- Substituting *Eqn. 6* into *Eqn. 4*, and into the 2nd term of *Eqn. 3*, we get

$$v\frac{dv}{dr} + \frac{RT}{\mu}\left(-\frac{1}{v}\frac{dv}{dr} - \frac{2}{r}\right) + \frac{GM_S}{r^2} = 0$$

$$\Rightarrow \left(v - \frac{RT}{\mu v}\right)\frac{dv}{dr} - \frac{2RT}{\mu r} + \frac{GM_S}{r^2} = 0$$

- A *critical point* occurs when *dv/dr* $\boxed{?}$ 0 i.e., when $\frac{2RT}{\mu r} = \frac{GM_S}{r^2}$

- Setting $\boxed{v_c = \sqrt{RT/\mu}}$ => $\boxed{r_c = GM_S/2v_c^2}$

Parker's solar wind (cont.)

- Rearranging => $\boxed{\left(v^2 - v_c^2\right)\frac{1}{v}\frac{dv}{dr} = 2\frac{v_c^2}{r^2}(r - r_c)}$ *Eqn. 7*

- Gives the momentum equation in terms of the flow velocity.

- If $r = r_c$, $dv/dr \rightarrow 0$ or $v = v_c$, and if $v = v_c$, $dv/dr \rightarrow \infty$ or $r = r_c$.

- An acceptable solution is when $r = r_c$ and $v = v_c$ (*critical point*).

- A solution to *Eqn. 7* can be found by direct integration:

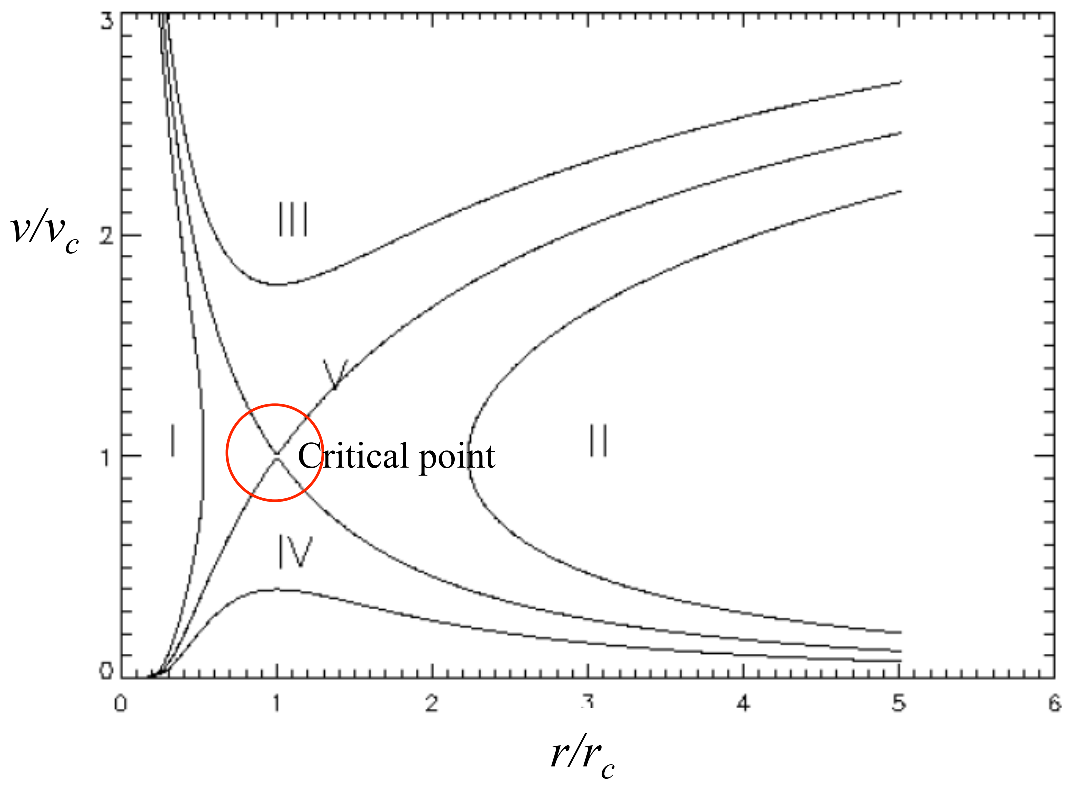
$$\boxed{\left(\frac{v}{v_c}\right)^2 - \ln\left(\frac{v}{v_c}\right)^2 = 4\ln\left(\frac{r}{r_c}\right) + 4\frac{r_c}{r} + C}$$

Eqn. 8
Parker's "Solar Wind Solutions"

where C is a constant of integration. Leads to five solutions depending on C.

Parker's solutions

- Solution I and II are double valued. Solution II also doesn't connect to the solar surface.
- Solution III is too large (supersonic) close to the Sun - not observed.
- Solution IV is called the solar breeze solution.
- Solution V is the solar wind solution (confirmed in 1960 by Mariner II). It passes through the critical point at $r = r_c$ and $v = v_c$.



Parker's solutions (cont.)

- Look at Solutions IV and V in more detail.

- Solution IV: For large r, $v \boxed{?}\, 0$ and *Eqn. 8* reduces to:

$$-\ln\left(\frac{v}{v_c}\right)^2 \approx 4\ln\left(\frac{r}{r_c}\right) \Rightarrow \frac{v}{v_c} = \left(\frac{r}{r_c}\right)^2$$

- Therefore, $r^2 v \boxed{?}\, r_c^{\,2} v_c = const$ or $v \approx \dfrac{1}{r^2}$

- From *Eqn. 5:* $\rho = \dfrac{const}{r^2 v} = \dfrac{const}{r_c^2 v_c} = const$

- From Ideal Gas Law: $P_\infty = R \boxed{?}_\infty T / \boxed{?} \Rightarrow P_\infty = const$

- The *solar breeze solution* results in high density and pressure at large r =>unphysical solution.

Parker's solutions (cont.)

- Solution V: From the figure, $v >> v_c$ for large r. *Eqn. 8* can be written:

$$\left(\frac{v}{v_c}\right)^2 \approx 4\ln\left(\frac{r}{r_c}\right) \Rightarrow v \approx v_c 2\sqrt{\ln\left(\frac{r}{r_c}\right)}$$

- The density is then: $\rho = \dfrac{const}{r^2 v} \approx \dfrac{const}{r^2\sqrt{\ln(r/r_c)}}$

  => $\boxed{?}$ $\boxed{?}$ 0 as r $\boxed{?}$ ∞.

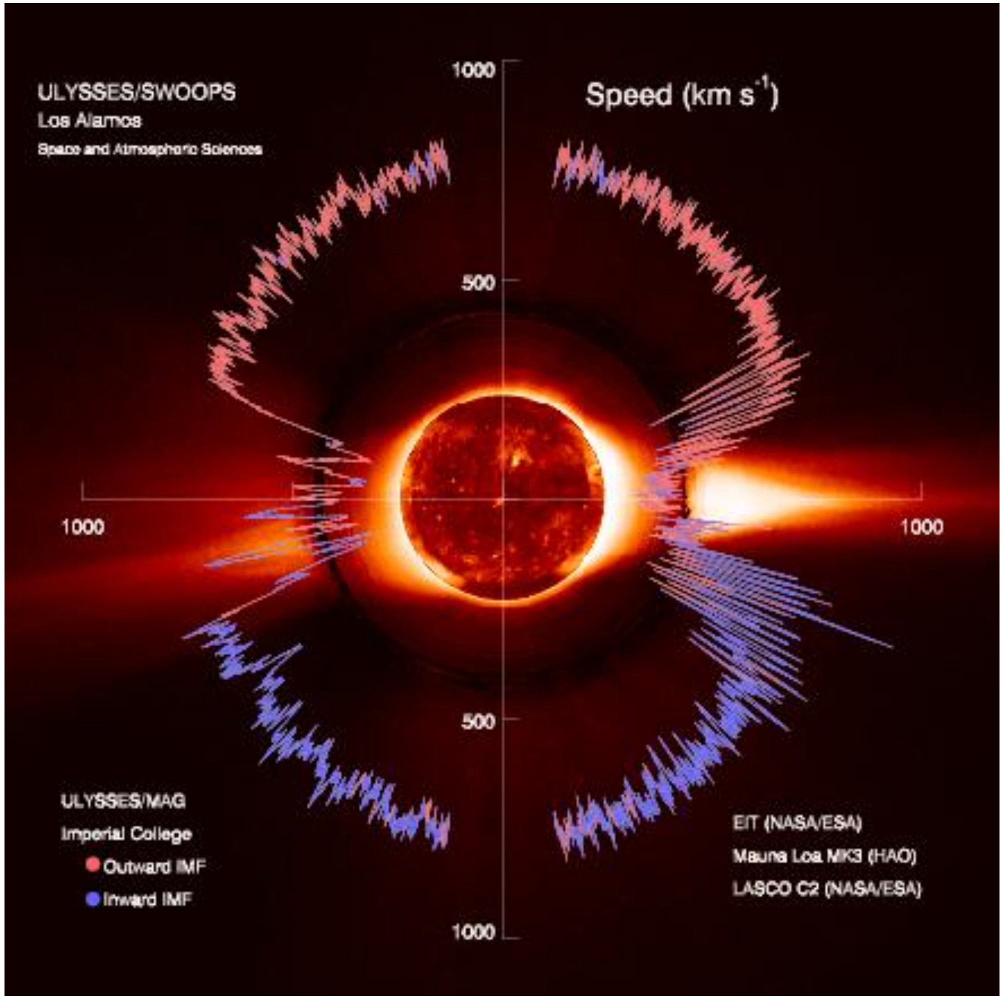
- As plasma is isothermal (i.e., $T = const.$), Ideal Gas Law => P $\boxed{?}$ 0 as r $\boxed{?}\infty$.

- This solution eventually matches interstellar gas properties => physically realistic model.

- Solution V is called the *solar wind solution*.

Observed solar wind

- Fast solar wind (>500 km s^{-1}) comes from coronal holes.
- Slow solar wind (<500 km s^{-1}) comes from closed magnetic field areas.
- Figure from McComas et al., Geophysical Research Letters, (2008).



Interplanetary magnetic field

- Solar rotation drags magnetic field into an Archimedian spiral ($r = a?$).

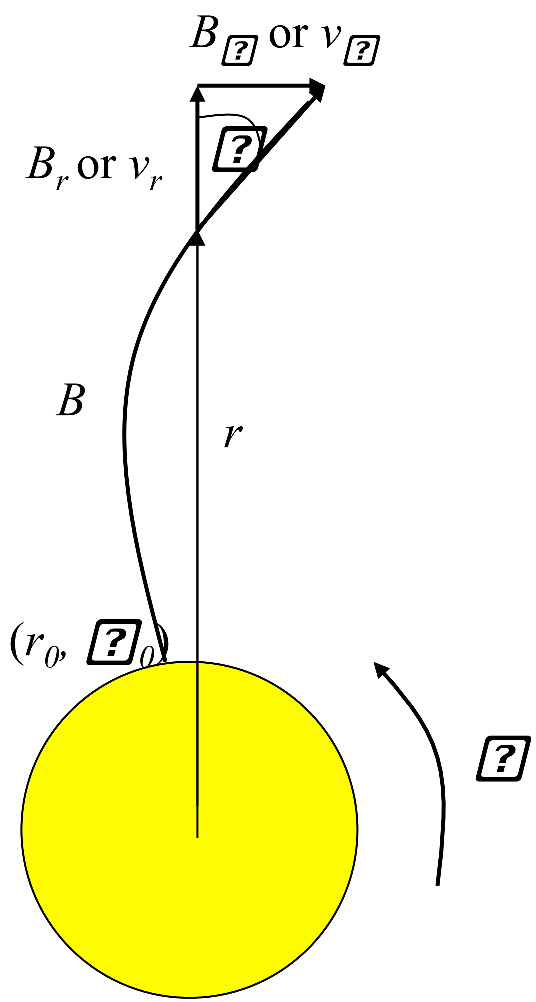
- Predicted by Eugene Parker => Parker Spiral:

$$r - r_0 = -(v/?)(? - ?_0)$$

- Winding angle:

$$\tan\psi = \frac{B_\phi}{B_r} = \frac{v_\phi}{v_r} = \frac{\Omega(r - r_0)}{v_r}$$

- Inclined at ~45º at 1 *AU* ~90º by 10 *AU*.



Alfven radius

- Close to the Sun, the solar wind is too weak to modify structure of magnetic field:

$$1/2\rho v^2 << \frac{B^2}{8\pi}$$

- Solar magnetic field therefore forces the solar wind to co-rotate with the Sun.

- When the solar wind becomes super-Alfvenic $$1/2\rho v^2 >> \frac{B^2}{8\pi}$$

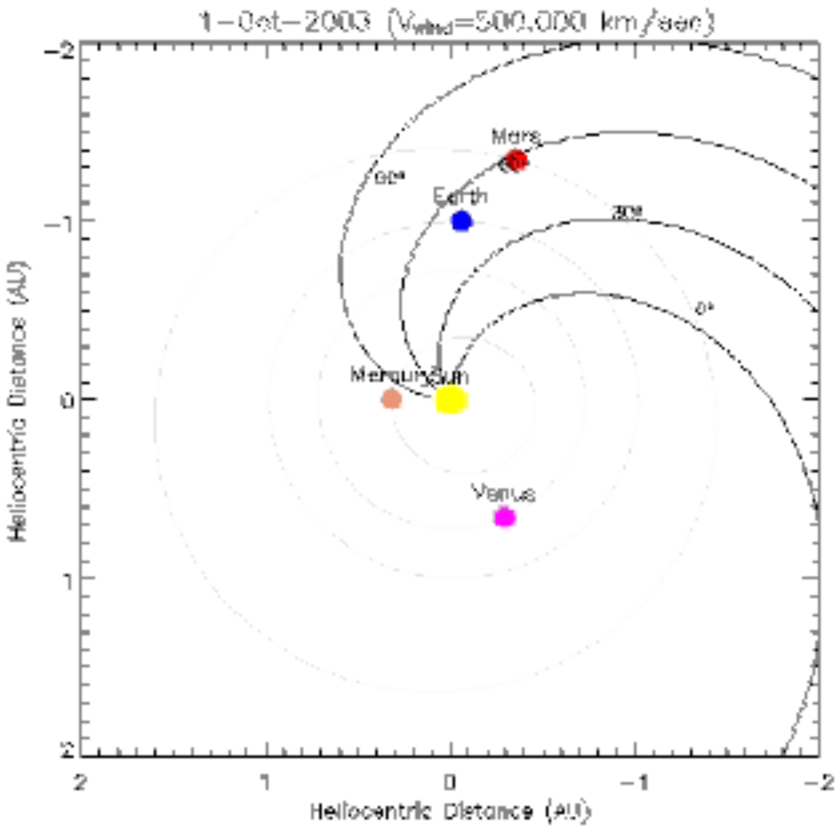
- This typically occurs at ~50 R_{sun} (0.25 AU).

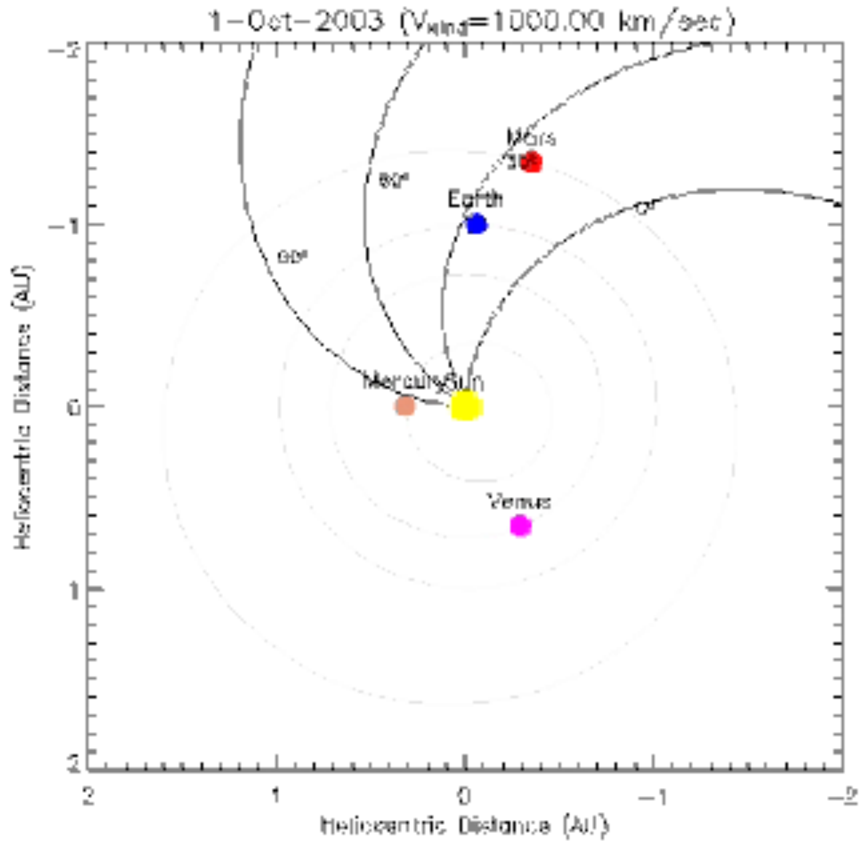
- Transition between regimes occurs at the *Alfven radius* (r_A), where $$1/2\rho v^2 = \frac{B^2}{8\pi}$$

- Assuming the Sun's field to be a dipole, $B = \frac{M}{r^3}$

$$\boxed{=> r_A = \left(\frac{M^2}{4\pi\rho v^2} \right)^{1/6}}$$

The Parker spiral





Heliosphere