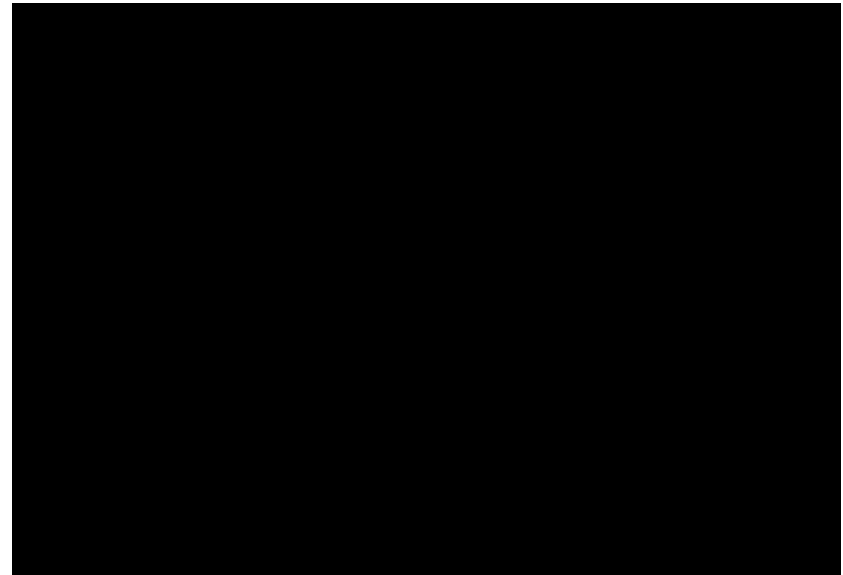# *Lecture 1-2: Properties of the Solar System*

o   Topics in this lecture:

    o   Planetary orbits

    o   Mass distribution

    o   Angular momentum
       distribution

Properties of the Solar System

1. Planets orbit roughly in the ecliptic plane.

2. Planetary orbits are slightly elliptical, and very nearly circular.

3. Planets and Sun revolve and orbit in a west-to-east direction. The planets obliquity (tilt of rotation axes to their orbits) are small. Uranus and Venus are exceptions.

4. The planets differ in composition. Their composition varies roughly with distance from the Sun: dense, metal-rich planets are in the inner part and giant, hydrogen-rich planets are in the outer part.

5. Meteorites differ in chemical and geologic properties from the planets and the Moon.

6. The rotation rates of the planets and asteroids are similar (5 to 15 hours).

7. Planet distances from the Sun obey Bode's law.

8. Planet-satellite systems resemble the solar system.

9. The Oort Cloud and Edgeworth-Kuiper Belt of comets.

10. Planets contain ~99% of the solar system's AM but Sun contains >99% of solar system's mass.

Orbits of the planets

o Planets moves around the Sun in an orbit effected by the Sun's mass, and to a less extent, by other bodies in the Solar System.

o Laws governing planetary motion was formulated by Johannes Kepler and based on Tycho Brahe's observations.

o Kepler's Laws:

1. Planets have elliptical orbits with the Sun at one focus.

2. As a planet orbits, a line connecting the planet to the Sun sweeps out equal areas in equal times.

3. The square of the orbital period is proportional to the cube of the semimajor axis of the orbit.
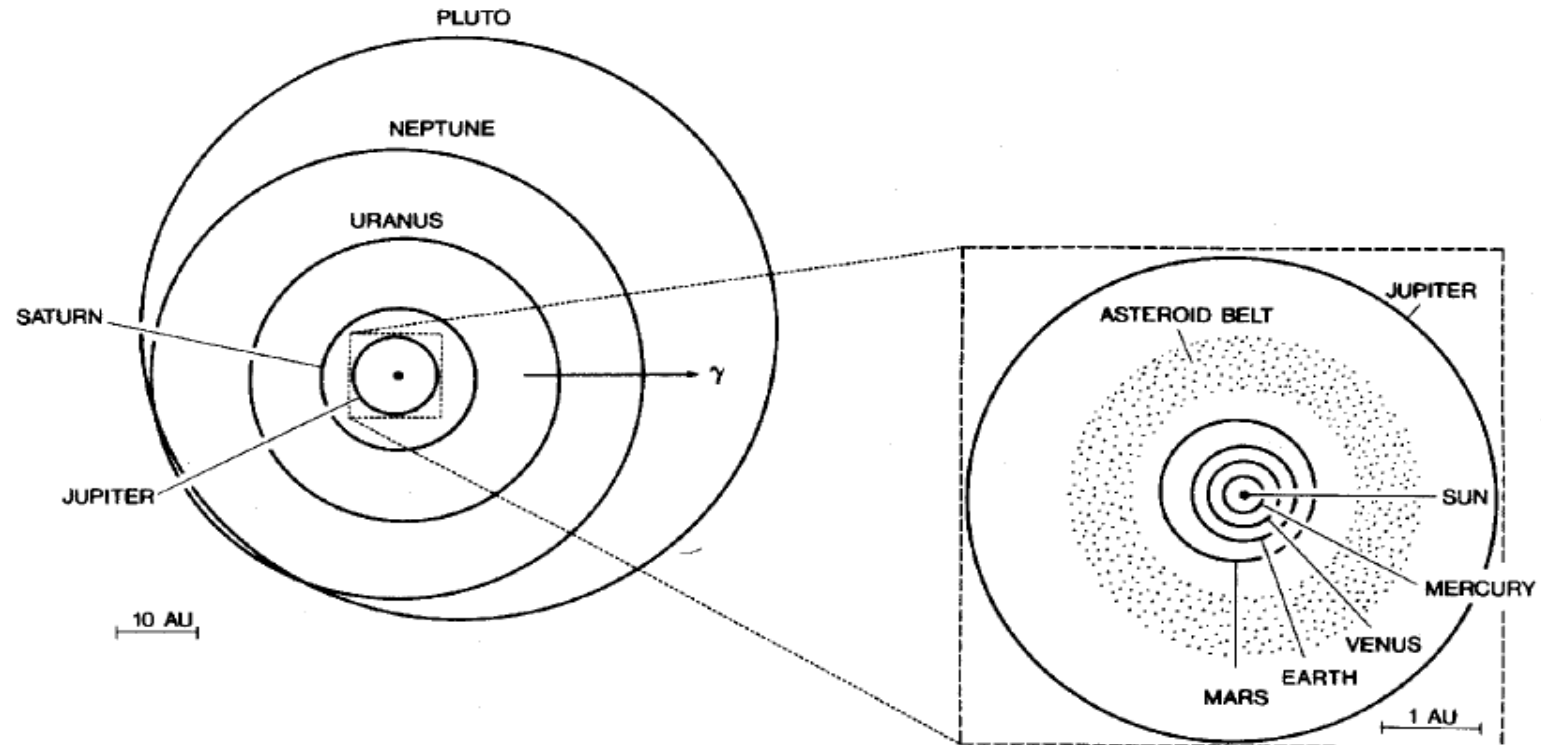
Orbits of Planets

Figure 1.1: The orbits of the planets about the Sun, projected onto the plane of the ecliptic. The left part of the figure shows the planets from Jupiter out to Pluto. The inner solar system, too small to be shown adequately with the outer planets, is expanded on the right. The orbits of Mars and Pluto are clearly not centred on the Sun. The position of the main asteroid belt is shown between Mars and Jupiter. The Kuiper belt (a second asteroid belt of icy asteroids), not shown, is outside the orbit of Neptune.

*From "Physical Processes in the Solar System" by J. Landstreet*

Planetary Properties

Class	Planet	a(A.U.)	M(M$_E$)	R(R$_E$)	ρ(g cm^{-3})
Terrestrials					
	Mercury	0.39	0.055	0.382	5.4
	Venus	0.72	0.815	0.949	5.2
	Earth	1	1	1	5.5
	Mars	1.52	0.107	0.532	3.9
Jovians					
	Jupiter	5.20	317.8	10.4-11.21	1.3
	Saturn	9.54	95.16	9.45	0.7
	Uranus	19.19	14.5	4.01	1.3
	Neptune	30.06	17.2	3.88	1.7

# Heliosphere (>100 AU from Sun)
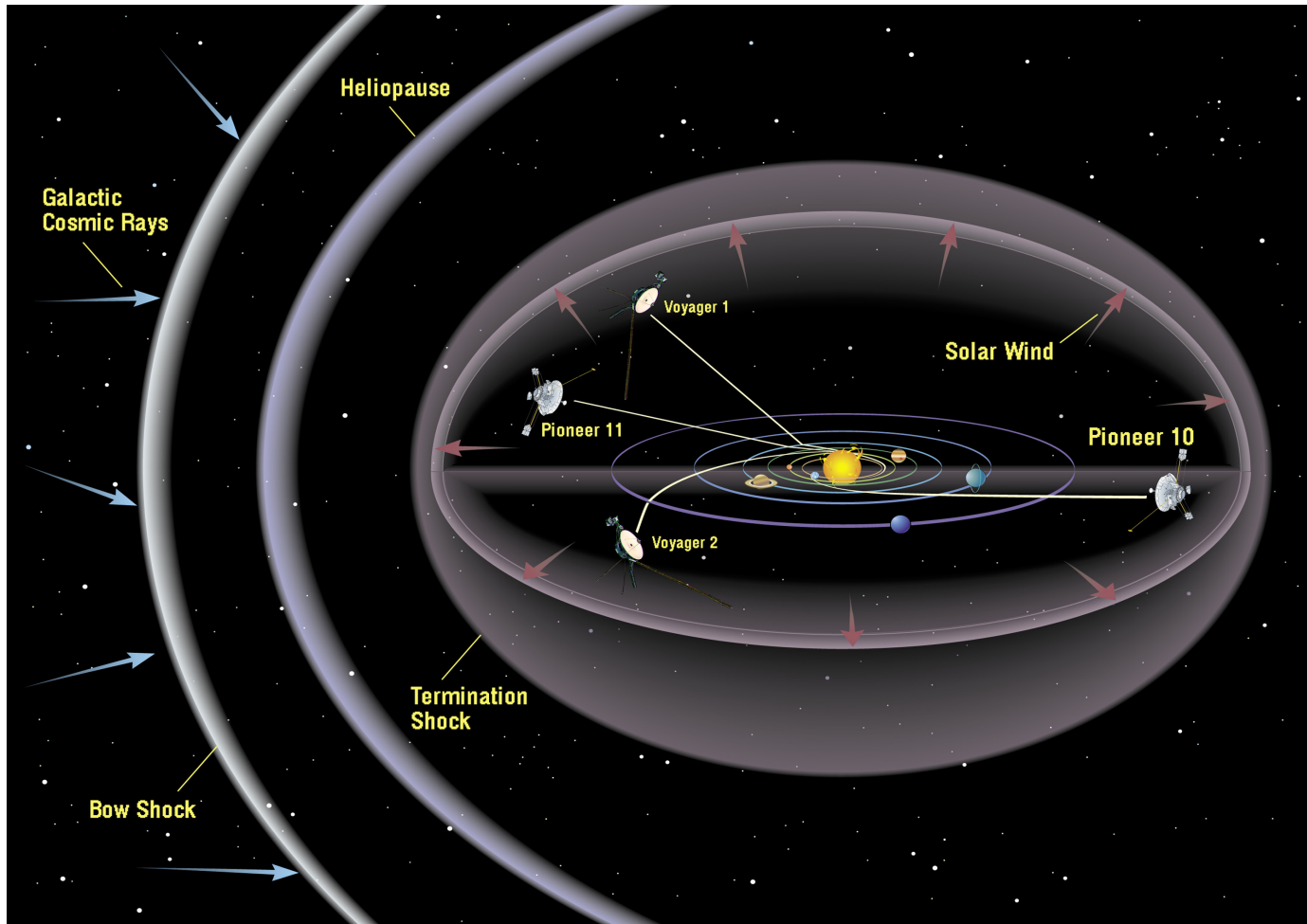
Kepler's 1st Law: Law of Orbits

o *Planets have elliptical orbits with the Sun at one focus.*

o Equation of ellipse: $r + r' = 2a$

o a is semimajor axis, b is semiminor axis of ellipse, F' and F are focal point.

o Distance of focus from ellipse centre is $a\,e$, where e is the eccentricity:
   o $e = 0$ => circle
   o $0 < e < 1$ => ellipse
   o $e = 1$ => parabola
   o $e > 1$ => hyperbola
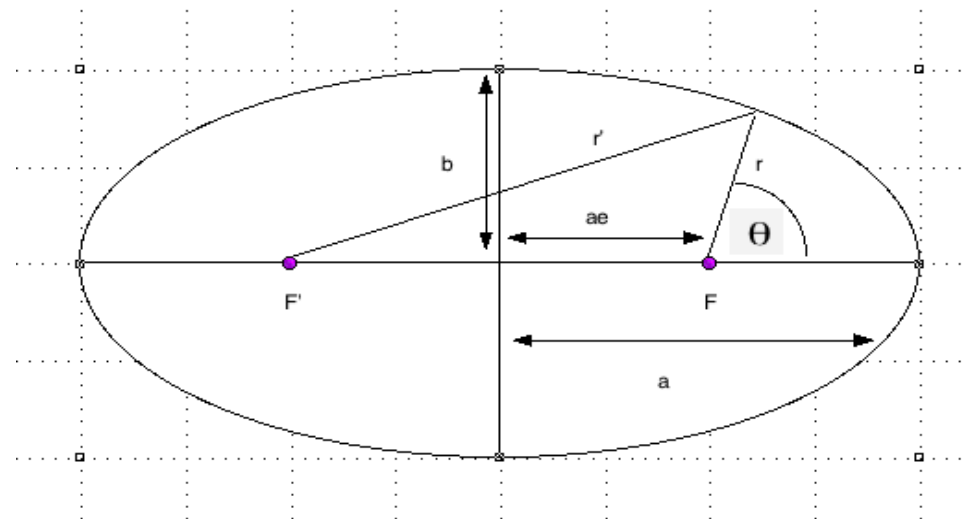
Kepler's 1st Law (cont.)

o   Implies that a planet's distance from the Sun varies during its orbit.

    o   Closest point to Sun: *perihelion.*

    o   Farthest point from Sun: *aphelion.*

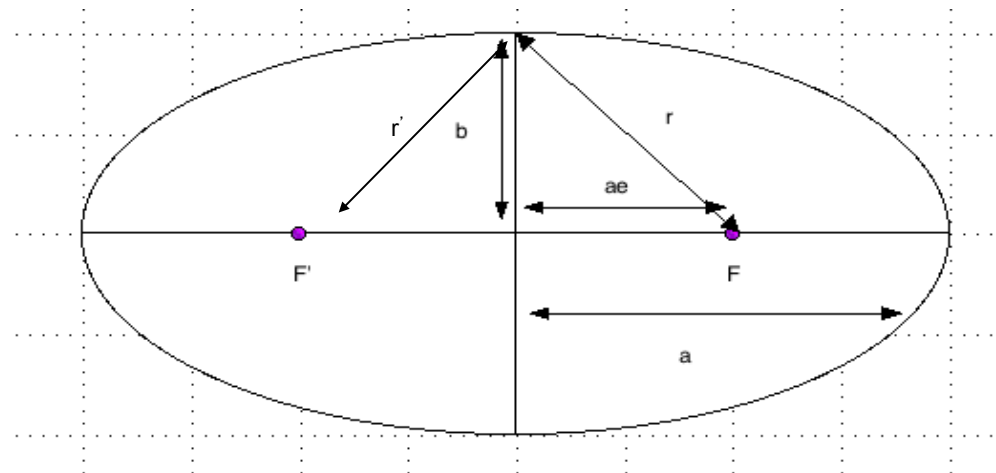   o   Average of perihelion and aphelion is called the *semimajor axis.*
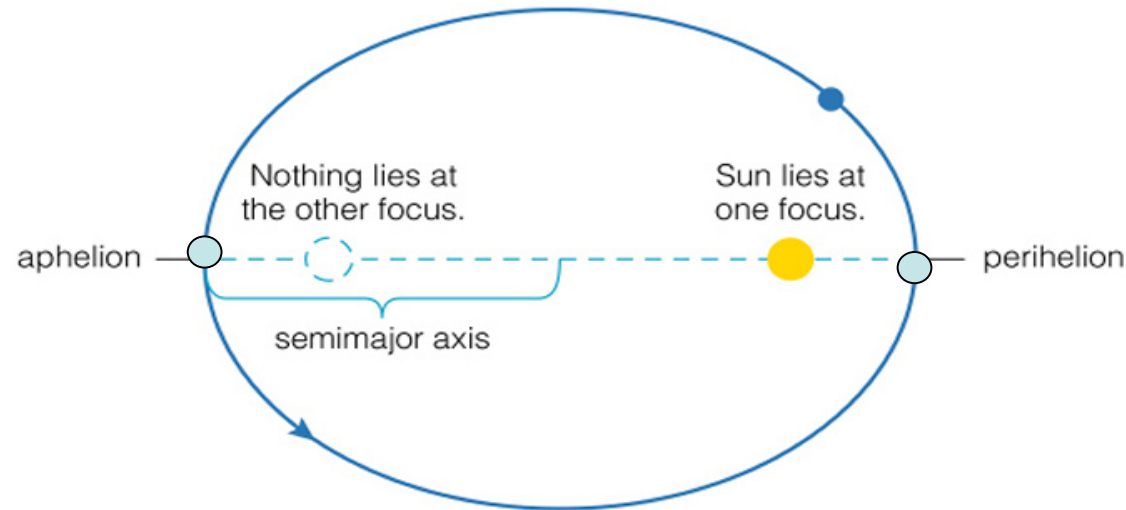
Elliptical orbits

o   Consider a point at either end of the semiminor axis where $r = r'$.

o   Using the Pythagorean Theorem, $r^2 = b^2 + (ae)^2$

o   Setting $r = a$, we may write:

$$a^2 = b^2 + a^2e^2$$
$$=> b^2 = a^2 (1 - e^2)$$

o   Relates semiminor axis
   to eccentricity and the
   semimajor axis.

Elliptical orbital path

o   From figure below, $r'^2 = r^2\sin^2\theta + (2ae + r\cos\theta)^2$ *Eqn 1*

o   But as $r + r' = 2a$ or $r' = 2a - r$, we may write $r'^2 = 4a^2 - 4ra + r^2$ *Eqn 2*

o   Equating the RHS of *Eqns 1* and *2*:

$$4a^2 - 4ra + r^2 = r^2\sin^2\theta + (2ae + r\cos\theta)^2$$
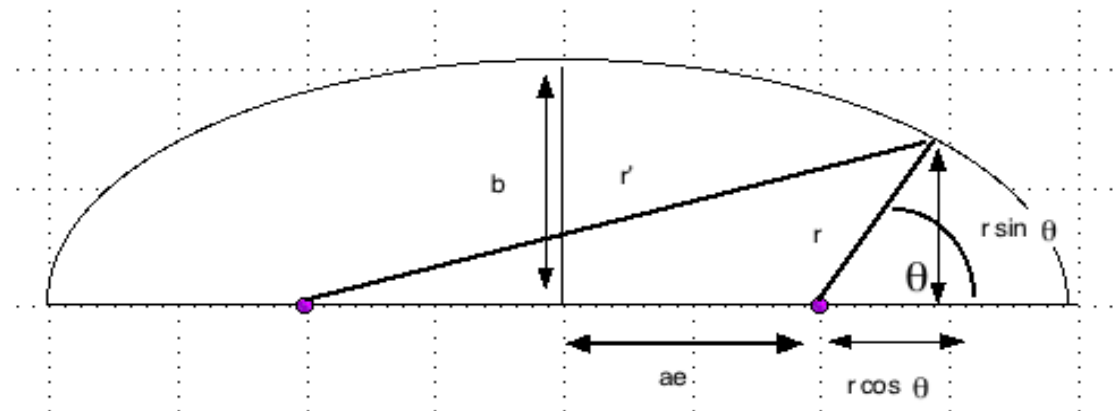$$= r^2(\sin^2\theta + \cos^2\theta) + 4a^2e^2 + 4aer\cos\theta$$

o   As $\sin^2\theta + \cos^2\theta = 1 \Rightarrow 4a^2 - 4ra = 4a^2e^2 + 4aer\cos\theta$

o   Rearranging gives,

$$\boxed{r(\theta) = \frac{a(1 - e^2)}{1 + e\cos\theta}}$$ *Eqn 3*

o   For *0<e<1,* this is the equation of an ellipse in polar coordinates.

Perihelion and aphelion distances

o   If $\theta = 0^o$, $cos\ \theta = 1$ and

$$r = \frac{a(1-e^2)}{1+e} = \frac{a(1-e)(1+e)}{(1+e)}$$

$$\Rightarrow r = a(1-e)$$

o   The planet is at *perihelion*, the closest point to the Sun.

o   If $\theta = 180^o$, $cos\ \theta = -1$ and
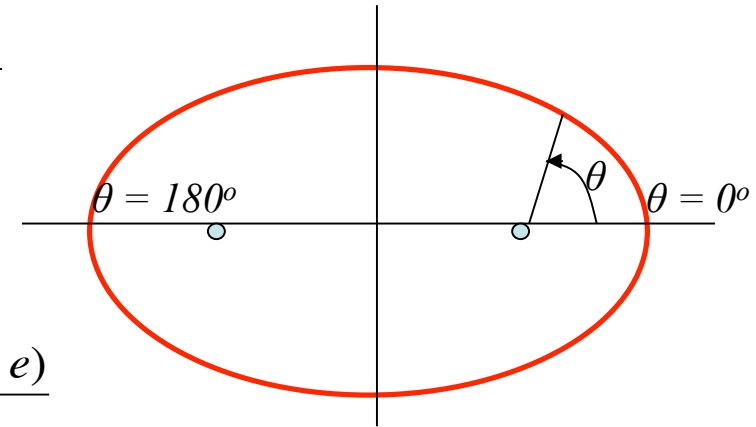
$$r = \frac{a(1-e^2)}{1-e} = \frac{a(1-e)(1+e)}{(1-e)}$$

$$\Rightarrow r = a(1+e)$$

o   The planet at the *aphelion*, the most distant point from the Sun.

o   Example: The semimajor axis of Mars is 1.5237 *AU* and the eccentricity is 0.0934. What is the distance of Mars at perihelion?

$$r = a(1-e)$$
$$= 1.5237(1-0.0934)$$
$$= 1.3814\ AU$$
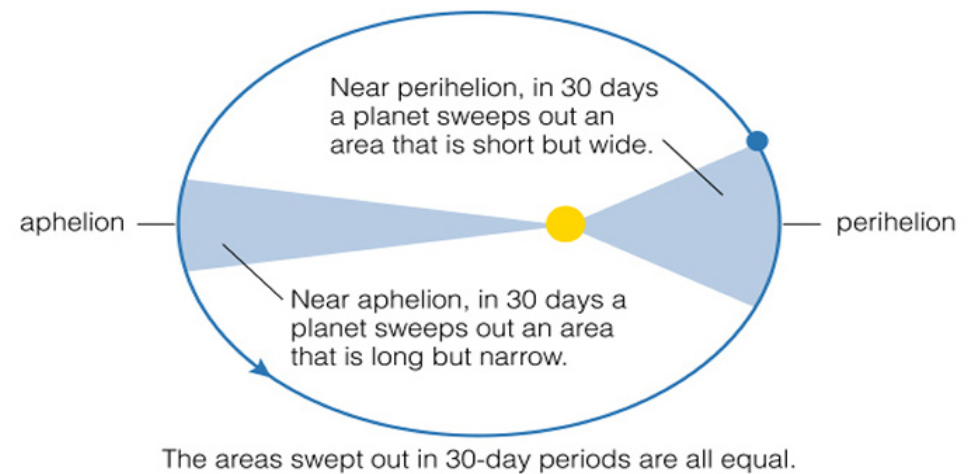
o   What is the distance of Mars at aphelion?

Kepler's 2nd Law: Law of areas

o *As a planet orbits, a line connecting the planet to the Sun sweeps out equal areas in equal times.*

$$\frac{dA}{dt} = const$$

=> Planet movies faster at perihelion.

Kepler's 2nd Law: Law of areas

o   Angular momentum of planet: $L = r \times p = m \, (r \times v)$.

o   During Δt, radius vector sweeps through $\Delta \theta = v_t \, \Delta t \, / \, r$, where v_t is the component of v perpendicular to r.

o   During this time, the radius vector has swept out the triangle, of area $A = r v_t \, \Delta t \, / \, 2$.

o   As $\Delta t -> 0$, $dA/dt = r v_t / 2 = 1/2 \, r^2 \, (d\theta/dt)$.

o   Now, the magnitude of L is given by
   $L = m \, v_t \, r = m \, r^2 \, d\theta/dt$.

   $\boxed{=> dA/dt = L \, / \, 2m = const}$

i.e. the rate of sweeping out area is a constant.
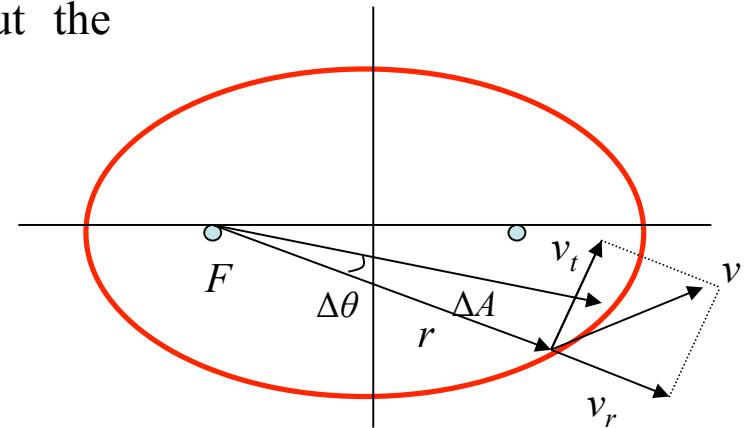
Kepler's 3rd Law: Law of Periods

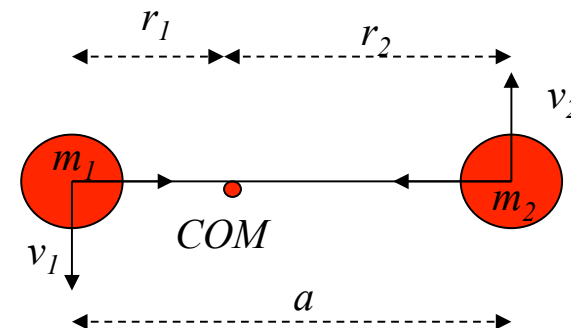o *The square of the orbital period is proportional to the cube of the semimajor axis of the orbit:*
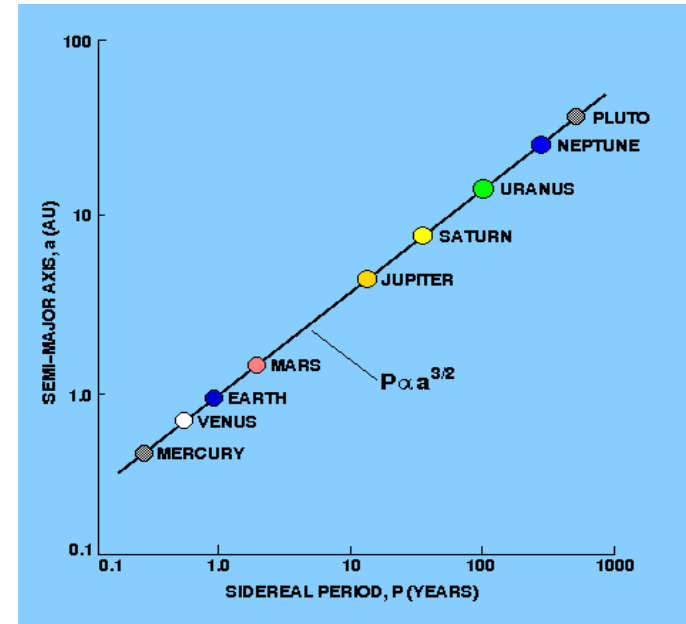
$$P^2 \sim a^3$$

o P is the period measured in years and a is the semimajor axis in *AU*.

o Consider m_1 and m_2 orbiting at r_1 and r_2. Both complete one orbit in period P. Forces due to centripetal accelerations are:

$$F_1 = m_1 v_1^2 / r_1 = 4\pi^2 \, m_1 r_1 / P^2$$
$$F_2 = m_1 v_2^2 / r_2 = 4\pi^2 \, m_2 r_2 / P^2$$

using $v = 2\pi r / P$

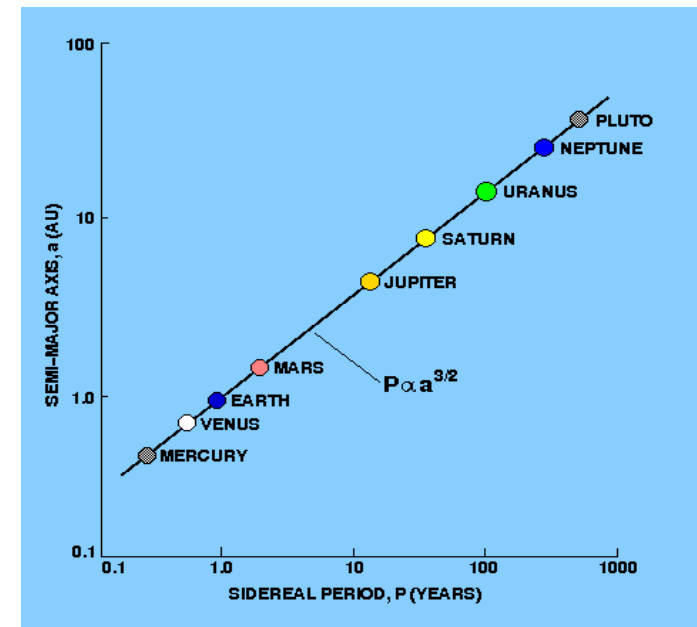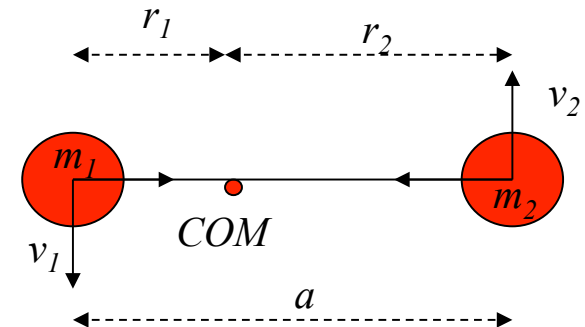# Kepler's 3rd Law: Law of Periods

o   As $F_1 = F_2 \Rightarrow r_1 / r_2 = m_2 / m_1$
(more massive body orbits closer to centre of mass).

o   Separation of the bodies is $a = r_1 + r_2$, and
$$r_1 = m_2 a / (m_1 + m_2)$$

o   Combining with F_1 and $F = F_1 = F_2 = Gm_1 m_2 / a^2$:

$$\boxed{P^2 = 4 \pi^2 a^3 / G(m_1 + m_2)}$$

o   As $M_{sun} (= m_1) >> m_{planet}(=m_2)$,
$const = 4 \pi^2 / GM_{Sun}$.

Bode's Law

o   Empirical prediction of planet distances from Sun.

o   Begin with:

       0, 3, 6, 12, 24, 48, 96, 192, 384

o   Now add 4:

       4, 7, 10, 16, 28, 52, 100, 196, 388

o   Then divide by 10:

      0.4, 0.7, 1.0, 1.6, 2.8, 5.2, 10.0, 19.6, 38.8

o   Sequence is close to mean distances of planets from the Sun.

o   *Bode's Law* or *Titus-Bode's Law*:

$$r_n = 0.4 + 0.3 \times 2^n$$

Planet	*Distance (AU)*	*Bode's Law (AU)*
Mercury	0.4	0.4
Venus	0.7	0.7
Earth	1.0	1.0
Mars	1.5	1.6
Ceres	2.8	2.8
Jupiter	5.2	5.2
Saturn	9.6	10.0
Uranus	19.2	19.6

Bode's Law (cont.)

Comparison of Bode's Law with Actual Distances

$$r_n = 0.4 + 0.3 \times 2^n$$
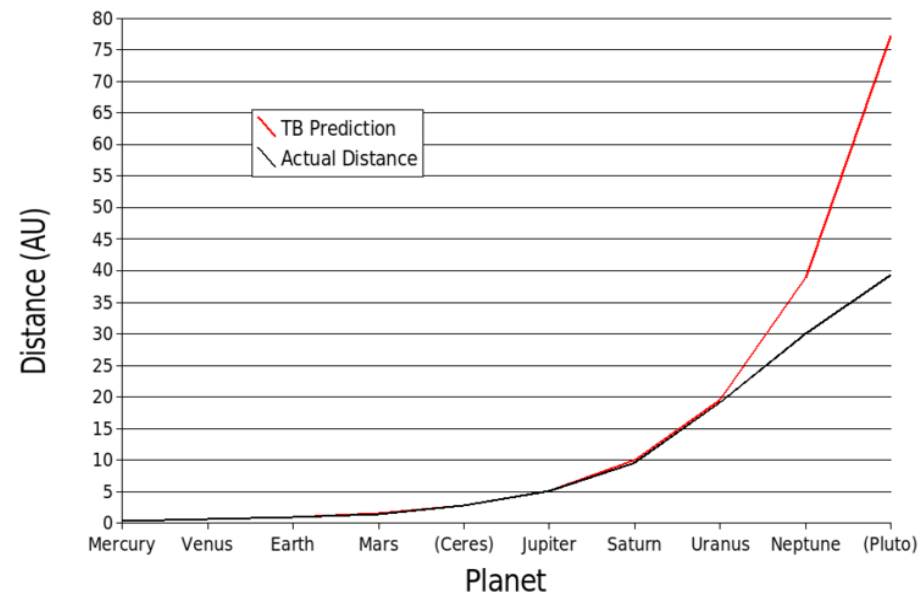
	n						
	0	1	2	3	4	5	6
	Planet						
	Mercury	Venus	Earth	Mars	Ceres	Jupiter	Saturn	Uranus
s-m axis	0.4	0.7	1.0	1.5	2.8	5.2	9.6	19.2
r_n	0.4	0.7	1.0	1.6	2.8	5.2	10.0	19.6

Bode's Law (cont.)

o Law lead Bode to predict existence of another planet between Mars and Jupiter - asteroids belt later found.

o Uranus fitted law when discovered.

o Neptune was discovered in 1846 at the position predicted by Adams, to explain the deviation of Uranus from its predicted orbit.

o Pluto's orbit when discovered in 1930 did not fit the relation. Not a "planet" anymore!

Planet	Distance (AU)	r_n (AU)
Uranus	19.2	19.6
Neptune	30.07	38.8
Pluto	39.5	77.2

The Solar System to scale
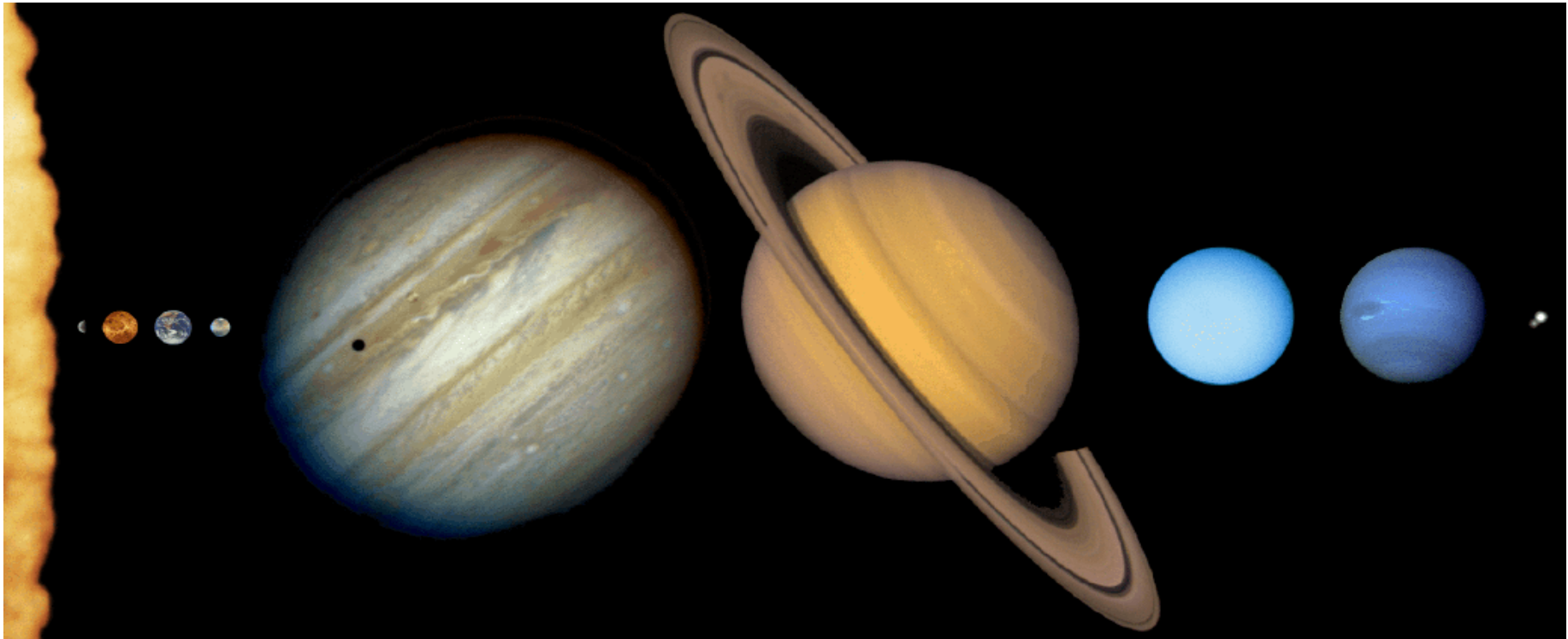
Mass distribution

o The density of a planet is measured in *g cm⁻³ (cgs units).*

$$Density = \frac{Mass}{Volume}$$

o Convenient because the density of water is *1 g cm⁻³.*

o To determine volume, need:

       1. Distance from Earth.
       2. Angular extent of the planet.
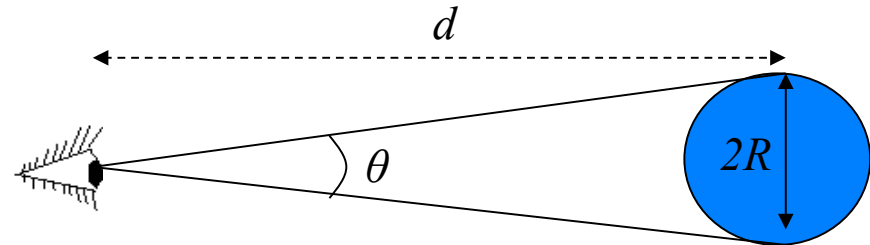
o To determine the mass (from Kepler's 3rd Law) we need:

       1. Mean Sun-planet separation.
       2. Orbital period.

Mass Distribution (cont.)

o The volume is determined from:

$$V = 4/3 \; \pi \; R^3$$

where $2R = 2\pi d\theta \; / \; 360$.
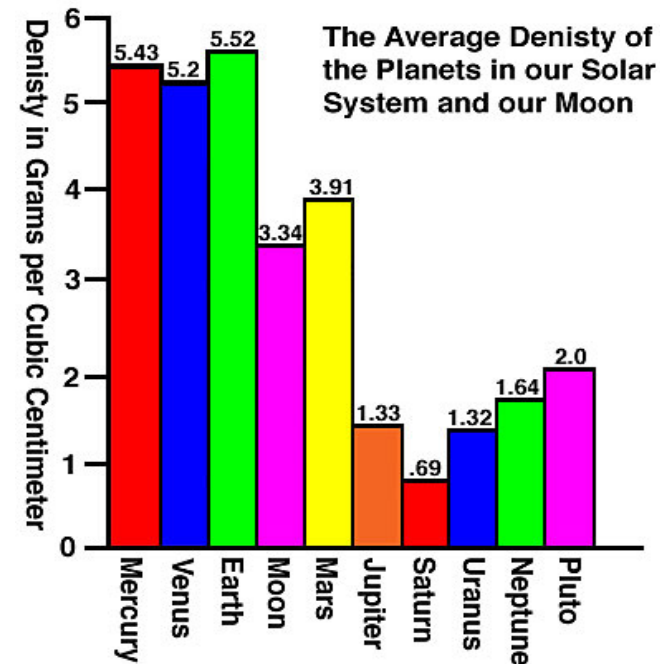
o Mass determined from Newton's form of Kepler's 3rd Law:

$$P^2 = 4 \; \pi^2 \; a^3 \; / \; G \; (m + M)$$
$$\Rightarrow m = (4 \; \pi^2 \; a^3 \; / \; G \; P^2) - M$$

o $\rho = m \; / \; V \quad g \; cm^{-3}$

o Compare to:
Cork:	0.2
Wood:	0.5
Water:	1.0
Basalt:	3.3
Lead:	11.0
Gold:	19.0

The Average Denisty of the Planets in our Solar System and our Moon

Denisty in Grams per Cubic Centimeter

Mercury 5.43, Venus 5.2, Earth 5.52, Moon 3.34, Mars 3.91, Jupiter 1.33, Saturn .69, Uranus 1.32, Neptune 1.64, Pluto 2.0

Properties of the planets

o   From consideration of size and density, can divide the planets into two
categories:

1. Terrestrial Planets

   o   Small size, high density and in the inner solar system.
   o   Mercury, Venus, Earth, Mars.

2. Jovian Planets

   o   Large size, low density and in outer solar system:
   o   Jupiter, Saturn, Uranus, Neptune.

o   **Pluto**

   o   Pluto is in category of own. It has small size and low density.

Compression vs. composition:
The inner planets

o   From their densities, inner planets likely to be composed of rock and some metal in cores.

o   Might expect that planets less massive than the Earth would have lesser densities, because they are less compressed at the center by gravity.

o   Amongst the terrestrial planets, this is *true* for both Mars and the Moon, which are both smaller *and* less dense than the Earth.

o   Venus is roughly the same size and density as Earth.

o   But, Mercury is both less massive *and* more dense than the Earth.
   => Has Mercury a different composition than the Earth?

Compression vs. composition:
The outer planets

o   What about the densities of the outer planets?

o   Might expect the outer planets, which are very massive, to be much more compressed than the inner planets, and so more dense.

o   In fact, these heavier bodies are *less* dense than the inner, terrestrial planets.

o   The only composition which we can use to construct such massive bodies with such low densities is a mixture of hydrogen and helium, the two lightest elements.

o   The composition of the outer planets is hence more similar to the Sun and stars than to the inner planets.

Angular momentum distribution

o The Sun has a relatively slow rotational period of ~26-days.

=> Like most G-, K- and M-class stars.

o The *orbital AM* is: $h_{orb} = mvr = 2\pi mr^2/P$ (r = distance)

o The *spin AM* of a inhomogeneous nonspherical rotating body is more difficult to evaluate.

o The mass of the body is $4\pi\rho_{ave}R^3/3$, where R is the radius and ρ_{ave} is the mean density.

Angular momentum of the Sun

o An average density adopted for Sun and it is assumed that mass is mostly within 0.6R. This is 0.72 for a perfect sphere, but the Sun is oblate.

o Mean density = 1.41 g cm^{-3}

o Mass = 2 x 10^{33} g

o Period at equator = 26.5 days = 2289600 s

o Radius = 6.96 x 10^{10} cm

o The spin AM is therefore:

$$h_{Sun} = mvR = \frac{4\pi\rho R^3}{3}\frac{2\pi r}{P}R$$

o Setting $R = 0.6R$ =>

$$h_{Sun} = mvR = \frac{4\pi\rho}{3}\frac{2\pi}{P}(0.6R)^5$$

$$= 1.9\rho R^5 / P = 2 \times 10^{48} \text{g cm}^2 \text{ s}^{-1}$$

o Detailed modelling gives ~1.7 x 10^{47} g cm^2 s^{-1}.

Angular momentum of the planets

o   The *orbital* AM is: $h_{orbit} = \dfrac{2\pi mr^2}{P}$

o   **Earth**
  o $m = 5.97 \times 10^{27}$ g
  o $r = 1\ AU = 1.5 \times 10^{13}$ cm
  o $P = 1$ year

$$h_{orbit} = \frac{2\pi(5.97 \times 10^{27})(1.5 \times 10^{13})^2}{365 \times 24 \times 60 \times 60}$$
$$= 2.6 \times 10^{47}\, \text{g cm}^2\ \text{s}^{-1}$$

o   **Jupiter**
  o $m = 1898 \times 10^{27}$ g
  o $r = 5.2\ AU = 5.2 \times 1.5 \times 10^{13}$ cm
  o $P = 11.86$ years

$$h_{orbit} = 1.9 \times 10^{50}\, \text{g cm}^2\ \text{s}^{-1}$$

o   **Saturn**
  o $m = 586 \times 10^{27}$ g
  o $r = 9.61$ AU
  o $P = 29.5$ years

$$h_{orbit} = 7.8 \times 10^{49}\, \text{g cm}^2\ \text{s}^{-1}$$

o   Jupiter therefore carries ~50% of the total AM of the Solar System, while the Jovian planets together make up ~99.27% of the total!
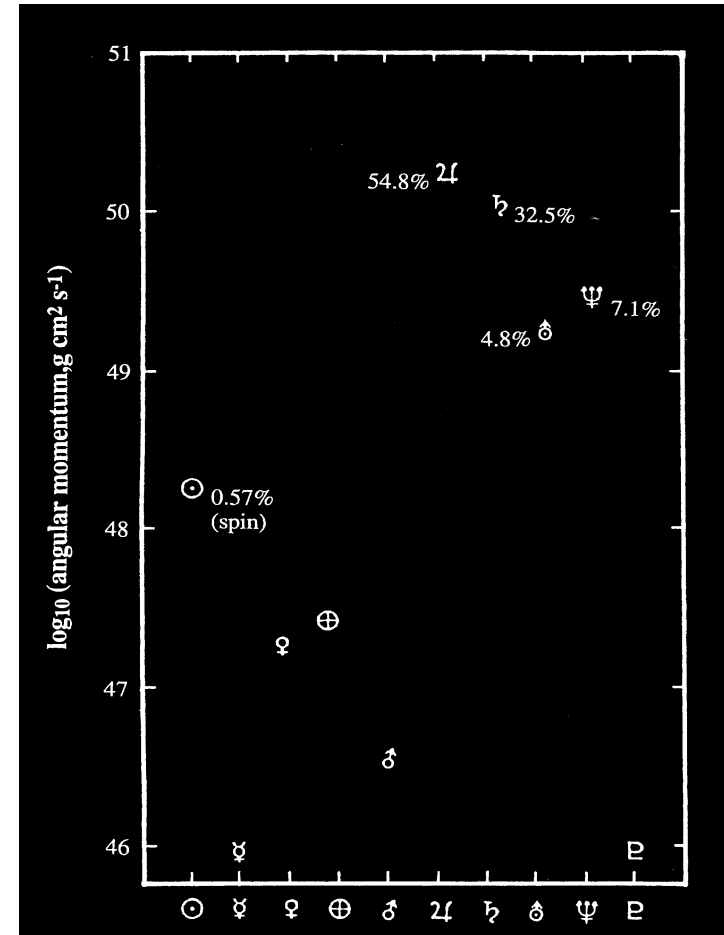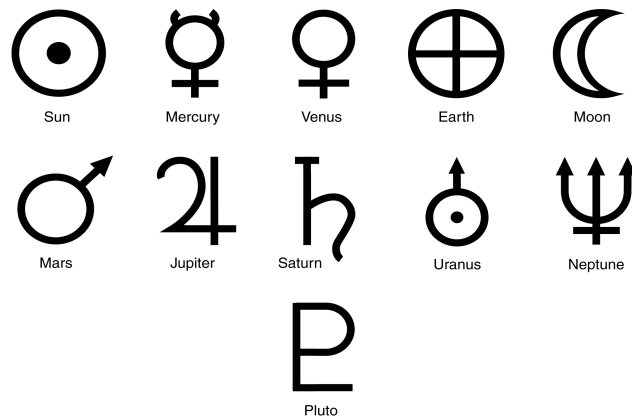
Angular momentum distribution

Planet	Mass (x10²⁷ kg)	Period (years)	AM (gcm²s⁻¹)	
Mercury	0.33	0.24	8.6×10^{45}	
Venus	4.87	0.61	1.9×10^{47}	0.4% AM
Earth	5.97	1	2.6×10^{47}	
Mars	0.64	1.88	3.4×10^{47}	
Jupiter	1898.8	11.86	1.9×10^{50}	
Saturn	568.41	9.5	7.8×10^{49}	
Uranus	86.97	19.31	1.7×10^{49}	99.2% AM
Neptune	102.85	30	2×10^{49}	
Pluto	0.0129	39.91	3.7×10^{45}	

o   =>Sun only has 0.4% of the total AM in solar system.

Orbital angular momenta of the planets

o Note the overwhelming importance of the Jovian planets.

o The symbol associated with each planet:

☉ Sun	☿ Mercury	♀ Venus	⊕ Earth	☽ Moon
♂ Mars	♃ Jupiter	♄ Saturn	⛢ Uranus	♆ Neptune
		♇ Pluto		

Mass and AM distributions

o   Although Sun contains 99.9% of the mass of Solar System, the outer planets have 98% of system's angular momentum.

o   This is a serious problem: material accreting onto the Sun cannot have retained all its original AM.

o   There are two parts to the problem:

1. How does material lose AM and fall into the star in the first place?

2. How does the star lose AM and slow down? Solar-type stars all rotate at about the same speed at the Sun.