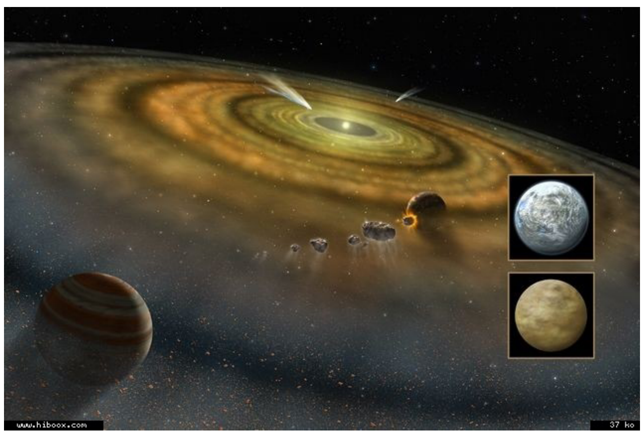
Lecture 3 - Minimum mass model of solar nebula

- Topics to be covered:
  - Composition and condensation
  - Surface density profile
  - Minimum mass of solar nebula

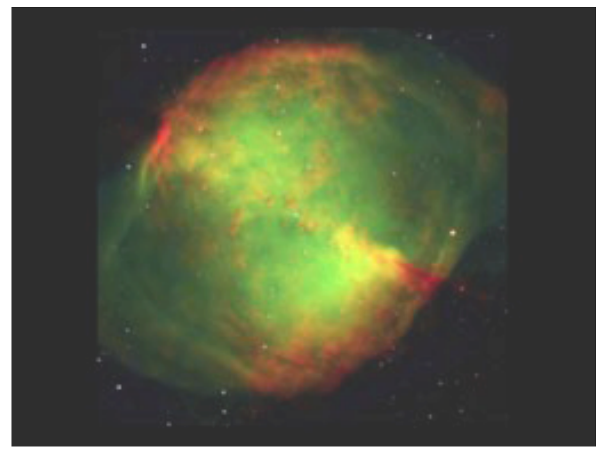
Minimum Mass Solar Nebula (MMSN)

- MMSN is *not* a nebula, but a protoplanetary disc.

Protoplanetary disk



Nebula



- Gives minimum mass of solid material to build the 8 planets.

Minimum mass of the solar nebula

- Can make approximation of minimum amount of solar nebula material that must have been present to form planets. Know:

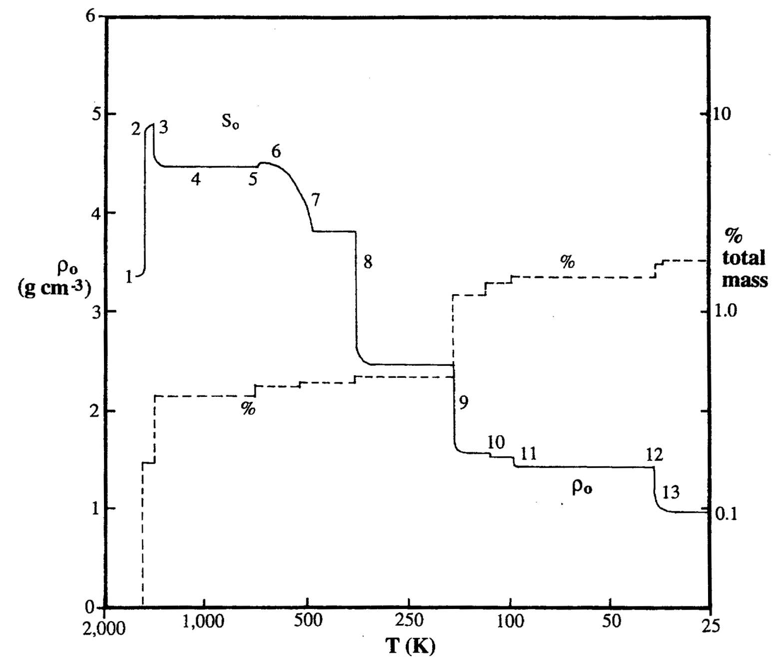
  1. Current masses, composition, location and radii of the planets.

  2. Cosmic elemental abundances.

  3. Condensation temperatures of material.

- Given % of material that condenses, can calculate minimum mass of original nebula from which the planets formed.

- Steps 1-8: metals & rock, steps 9-13: ices



• Figure from Page 115 of "Physics & Chemistry of the Solar System" by Lewis

Nebula composition

- Assume solar/cosmic abundances:

Representative elements	**Main nebular Low-T material**	**Fraction of nebular mass**
H, He	Gas H_2, He	98.4 %
C, N, O	Volatiles (ices) H_2O, CH_4, NH_3	1.2 %
Si, Mg, Fe	Refractories (metals, silicates)	0.3 %

Minimum mass for terrestrial planets

- ***Mercury:***~5.43 *g cm*$^{-3}$ => complete condensation of Fe (~0.285% M_{nebula}).

  $$0.285\%\ M_{nebula} = 100\ \%\ M_{mercury}$$
  $$=> M_{nebula} = (100/\ 0.285)\ M_{mercury}$$
  $$= 350\ M_{mercury}$$

- ***Venus:*** ~5.24 *g cm*$^{-3}$ => condensation from Fe and silicates (~0.37% M_{nebula}).

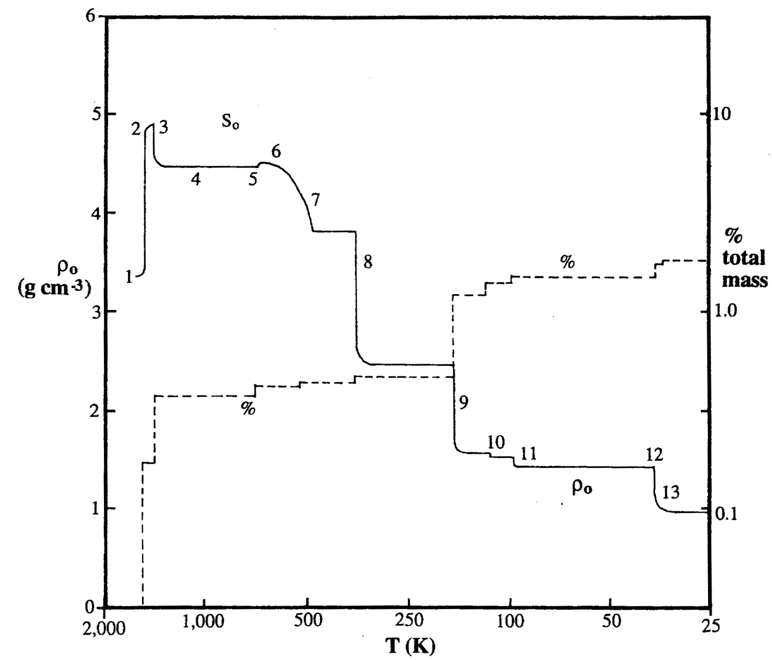
  $$=>(100\%\ /\ 0.37\%\ )\ M_{venus} = 270\ M_{venus}$$

- ***Earth/Mars:*** 0.43% of material condensed at cooler temperatures.

  $$=> (100\%\ /\ 0.43\%\ )\ M_{earth} = 235\ M_{earth}$$

- ***Asteroids:*** Cooler temperatures produce more condensation ~ 0.5 %.

  $$=> (100\%\ /\ 0.5\%) = 200\ M_{asteroids}$$



Minimum mass for terrestrial planets

- What is the minimum mass required to make the Terrestrial planets?

Planet	Factor	Mass ($x10^{26}$ *g*)	Min Mass ($x10^{26}$ *g*)
Mercury	350	3.3	1155
Venus	270	48.7	13149
Earth	235	59.8	14053
Mars	235	6.4	1504
Asteroids	200	0.1	20

- Total of the 4^{th} column is 29881×10^{26} *g*. This is the minimum mass required to form the Terrestrial planets =>2.9881×10^{30} *g* ~ 500 M_{earth}.

Minimum mass for jovian planets and pluto

- ***Jupiter:*** Almost nebula composition due to gas capture ~20%.
  $\Rightarrow M_{nebula} = 100 / 20\ M_{jupiter} \sim 5\ M_{jupiter}$ is minimum mass required.

- ***Saturn:*** Cooler than Jupiter, with slightly different composition ~12.5%.
  $\Rightarrow M_{nebula} = 100/12.5\ M_{saturn} \sim 8\ M_{saturn}$

- ***Uranus:*** Less gas capture ~6.7% condensed to form planet.
  $\Rightarrow M_{nebula} = 100/6.7\ M_{uranus} = 15\ M_{uranus}$

- ***Neptune:*** ~5% of solar nebula material condensed to form planet.
  $\Rightarrow M_{nebula} = 100/5\ M_{neptune} = 20\ M_{neptune}$

- ***Pluto:*** Main fraction due to ices ~1.4 % $\Rightarrow M_{nebula} = 100/0.14\ M_{pluto} = 70\ M_{pluto}$

Minimum mass for jovian planets

- What is the minimum mass required to make the Jovian planets?

Planet	Mass ($\times 10^{26}$ *g*)	Factor	Min Mass ($\times 10^{26}$ *g*)
Jupiter	19040	5	95200
Saturn	5695	8	55560
Uranus	890	15	13050
Neptune	1032	20	20640

- Total mass is therefore = 184450 x 10^{26} g = 3085 M_{earth}.
- This is minimum solar nebula mass required to make the Jovian planets.

Minimum nebula mass

- The minimum mass required to condense the nine planets is therefore:

Planet	***M* (x M_{earth})**
Terrestrial	500
Jovian	3085
Pluto	0.119
	3585 M_{earth}

- This is the minimum mass required to produce the planets.

- As $M_{sun} \sim 2 \times 10^{33}\ g$, the mass required to make the planets is therefore ~0.01 M_{sun}.

- Disk contained 1/100 of the solar mass.

Nebular surface density profile

- To make a more precise estimate, distribute min mass requirements over series of annuli, centred on each planet.

- Choose boundaries of annuli to be halfway between the orbits of each planet. i.e., Mercury @ 0.38 *AU* and Venus @ 0.72 *AU* = > (0.72-0.38)/2 = 0.17 *AU*.

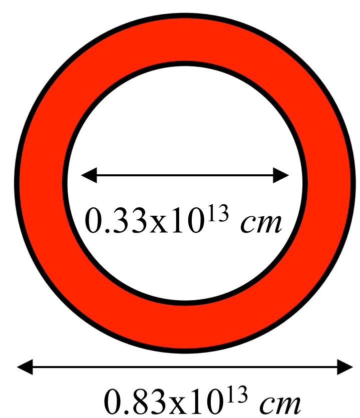
- We therefore estimate that Mercury was formed from material within an annulus of 0.38±0.17 *AU* => 0.33 - 0.83 x 10^{13} *cm*.

- The surface density of an annulus, ? = *mass / area,*

  where *area* = ? r_{outer}^2 - ? r_{inner}^2

  = ? $[(0.83 \times 10^{13})^2 - (0.33 \times 10^{13})^2]$

  = 1.82 x 10^{26} cm^2



- Surface density of disk near Mercury is therefore:

  $1160 \times 10^{26} / 1.82 \times 10^{26} = 637\ g\ cm^{-2}$

Nebular surface density profile

- For Venus at 0.72 *AU,* Mercury is at 0.38 *AU* and Earth is at 1 *AU* => Venus' annulus extends from

  (0.72 - 0.38)/2 = 0.17 to (1 - 0.72)/2 = 0.14

- The material that formed Venus was located between 0.72 - 0.17 *AU* and 0.72 + 0.14 or 0.55-0.86 *AU*. This is 0.83-1.29 x 10^{13} *cm*.

- Area is then = $\boxed{?}\, r_{outer}^{2} - \boxed{?}\,_{inner}^{2} = 3.06 \times 10^{26}\ cm^{2}$.

  $=> \boxed{?} = 13150 \times 10^{26} / 3.06 \times 1\ 10^{26} = 4300\ g\ cm^{-2}$.

- This is the approximate surface density of the disk where Venus formed.

- For Jupiter at 5.2 *AU,* the Asteroids are at 3 *AU* and Saturn is at 9.6 *AU*. The annulus therefore ranges from 4 - 7.2 or 6 - 11 x 10^{13} *cm*.

- As the area = 267 x 10^{13} cm^{2} => $\boxed{?} = 95200 \times 10^{26} / 267 \times 10^{26} = 356\ g\ cm^{-2}$

Minimum mass and density

Table IV.8 Minimum Mass of the Primitive Solar Nebula

Planet	Mass (10^{26}g)	F^a	M_{solar} (10^{26}g)	r_{ann} (10^{13}cm)	A_{ann}(cm^2) (x10^{-26})	$\sigma = M/A$ (g cm^{-2})
Mercury	3.3	350	1,160	0.33–0.83	1.82	637
Venus	48.7	270	13,150	0.83–1.29	3.06	4300
Earth	59.8	235	14,950	1.29–1.89	6.00	2500
Mars	6.4	235	1,504	1.89–3.20	20.95	72
Asteroids	0.1	200	20	3.2–6.0	80.9	0.25
Jupiter	19,040	5	95,200	6.0–11.0	267	355
Saturn	5,695	8	55,560	11.0–21.5	1072	42.4
Uranus	870	15	13,050	21.5–36.8	2802	4.7
Neptune	1,032	20	20,640	36.8–52.0	4240	4.9
Pluto	0.1	70	7	52–70	6900	0.001

[a] *F* is the factor by which the planetary mass must be multiplied to adjust the observed material to solar composition.

From "Physics and Chemistry of the Solar System" by Lewis

Surface density of solar nebula

- Surface density of the drops off as:

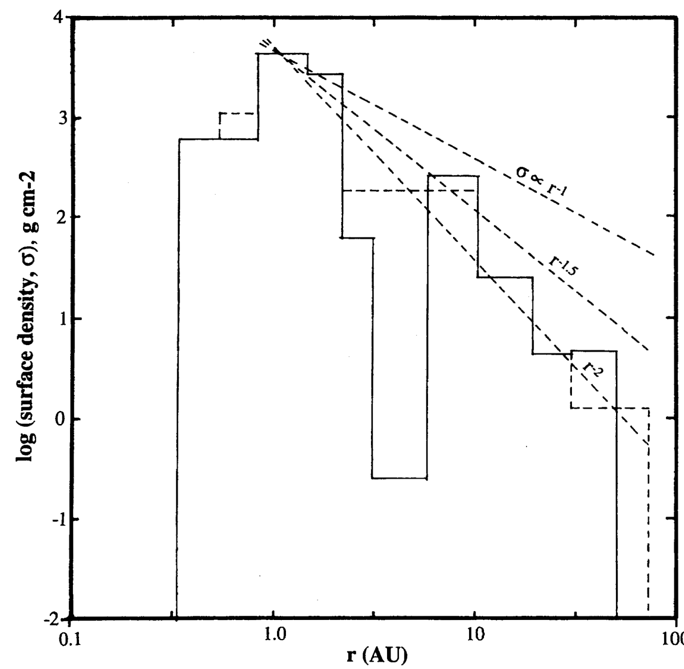
$$?(r) = ?_0 \, r^{-?}$$

- $? \sim 1.5$ and $?_0 \sim 3{,}300 \; g \; cm^{-3}$.

- Local deficit of mass in asteroid belt. Mars is also somewhat deficient in mass.

- Inside Mercury's orbit, nebula material probably cleared out by falling in on Sun or blown out.

- Outer edge may be due to a finite scale size of the original nebular condensation.



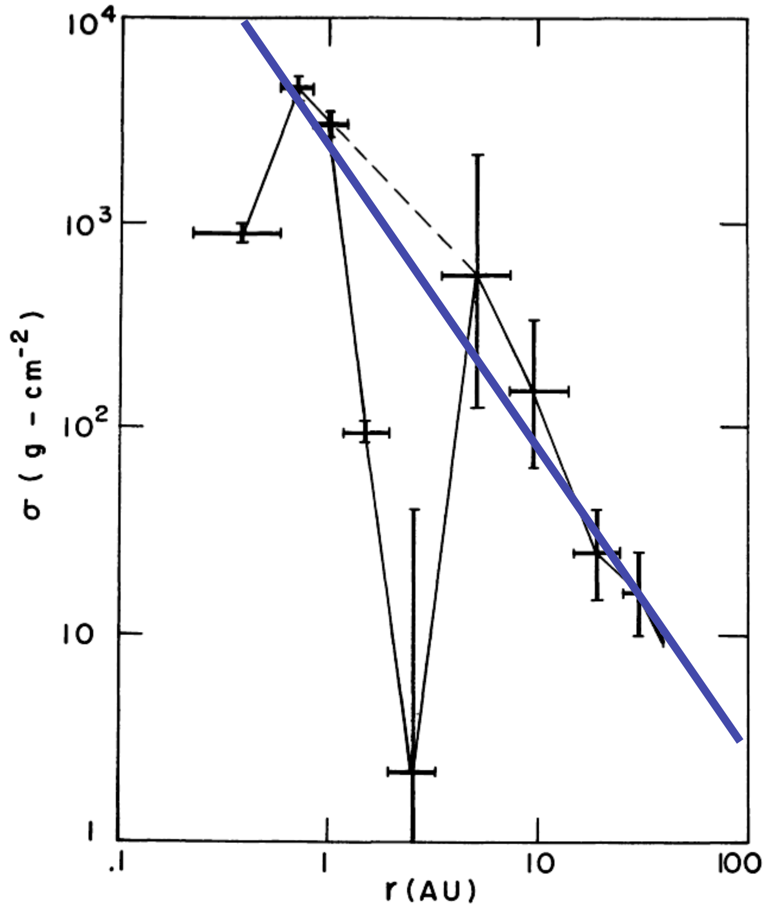
Surface density of solar nebula

- Hayashi (1981) widely used:

  $\sigma(r) = 1700\,(r\,/\,1\mathrm{AU})^{-3/2}$ g cm^{-2}

- Weidenschilling (1977) produced figure at right which shows similar trend.

- Mars and asteroids appears to be under-dense.



Minimum Mass Extrasolar Nebula

- Chuan & Laughlin (2013) used 1,925 exoplanets observed by *Kepler* (http://arxiv.org/abs/1211.1673)

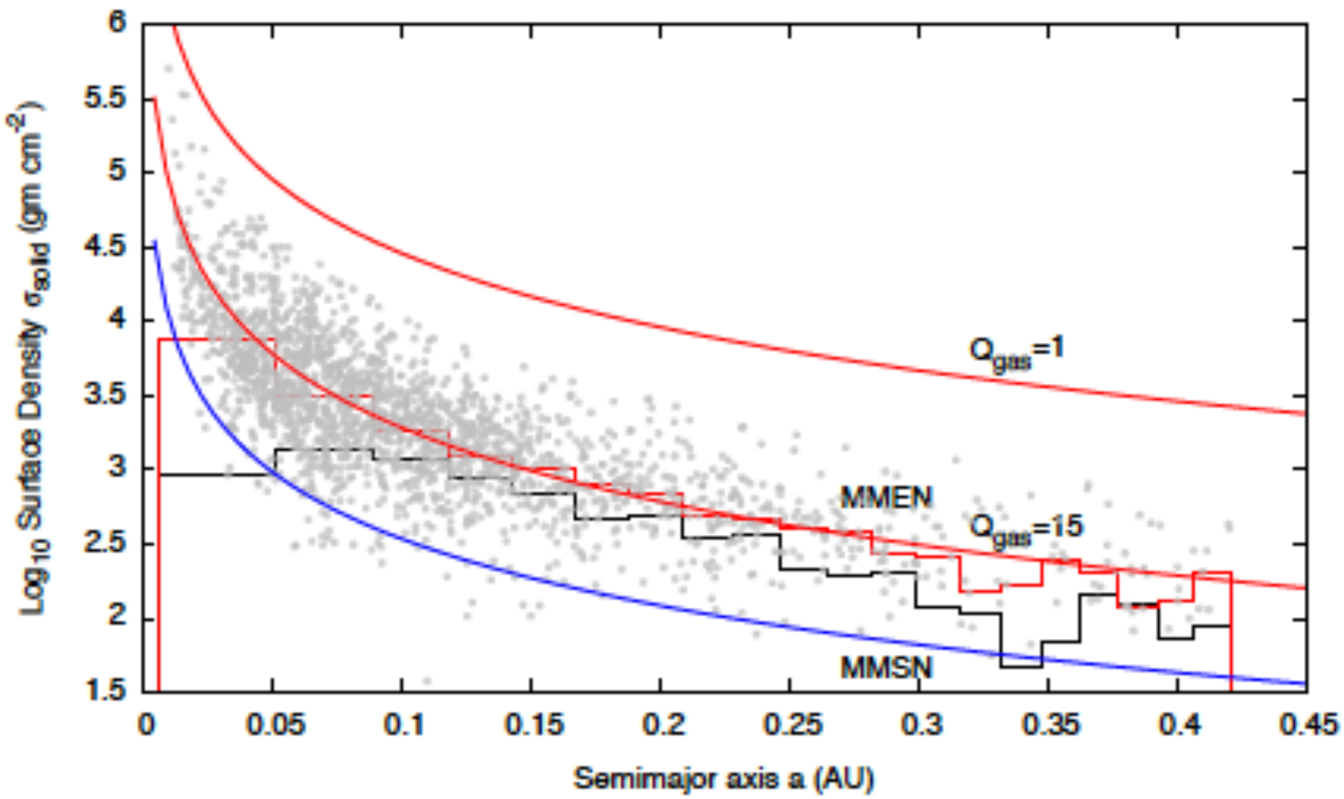
- Surface density near exoplanet is: $$\sigma_{exoplanet} = \frac{M_{exoplanet}}{2\pi a_{exoplanet}^2}$$

  where $M_{exoplanet} = \left(R_{exoplanet} / R_{Earth}\right)^{2.06} M_{Earth}$

  is best fit power-law relationship for the Solar System.

- Semimajor axis from Kepler's laws is: $a_{exoplanet} = \left(P / yr\right)^{2/3}$ AU

Minimum Mass Extrasolar Nebula



- MMEN: $\sigma = 620\left(\frac{a}{0.2AU}\right)^{-1.6}$

Minimum mass estimate

- Can also estimate minimum mass from [?] :

$$M = \int \sigma(r) dA = \int_0^{2\pi} \int_{R_S}^{R_F} \sigma(r) r dr d\theta$$

where R_S is the radius of the Sun and R_F is the max distance of Pluto.

- Assume that [?]$(r) = 3300 (r / R_E)^{-2}$, where $R_E = 1\ AU$. Therefore,

$$M = \int_0^{2\pi} \int_{R_S}^{R_F} (3300(r/R_E)^{-2}) r dr d\theta$$

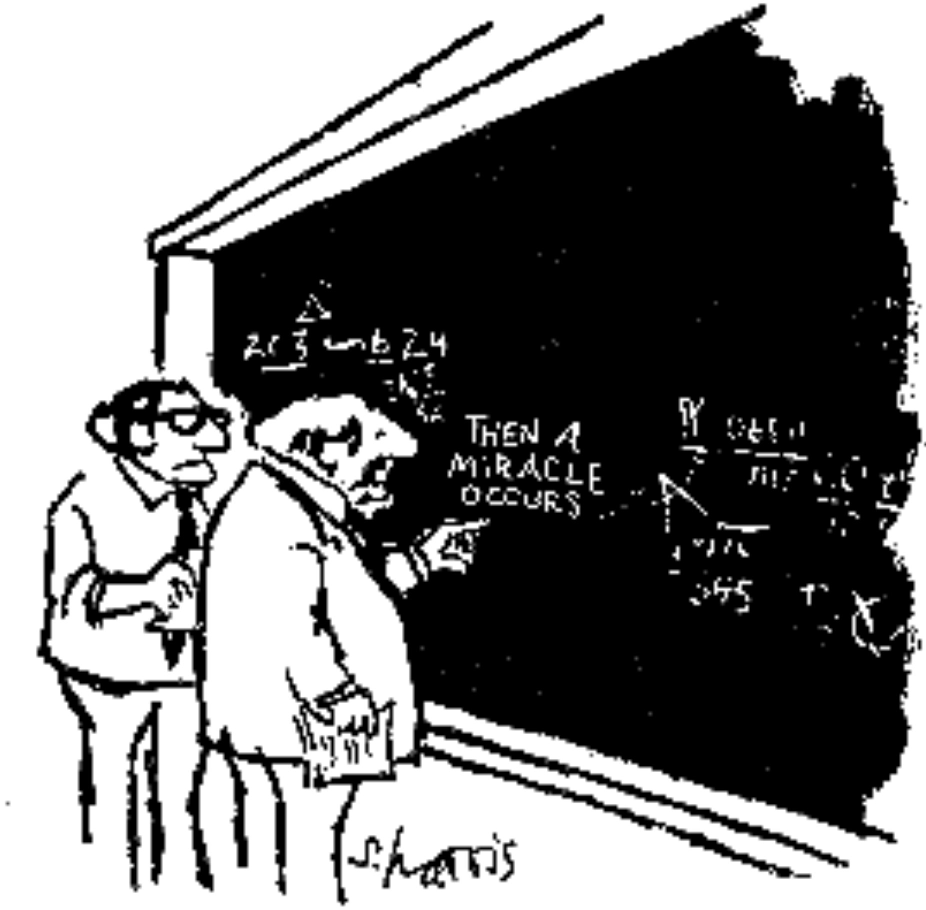
$$= 3300 R_E^2 \int_0^{2\pi} d\theta \int_{R_S}^{R_F} r^{-1} dr d\theta$$

$$= 6600 \pi R_E^2 \ln(R_F / R_S)$$

- Setting $R_E = 1.49 \times 10^{13}$ cm, and $R_S = 6.96 \times 10^{10}$ cm, and $R_F = 39$ AU =>

M [?] $0.02\ M_{Sun}$

- Ie approximately a factor of two of previous estimate.



"I think you should be more explicit here in step two."