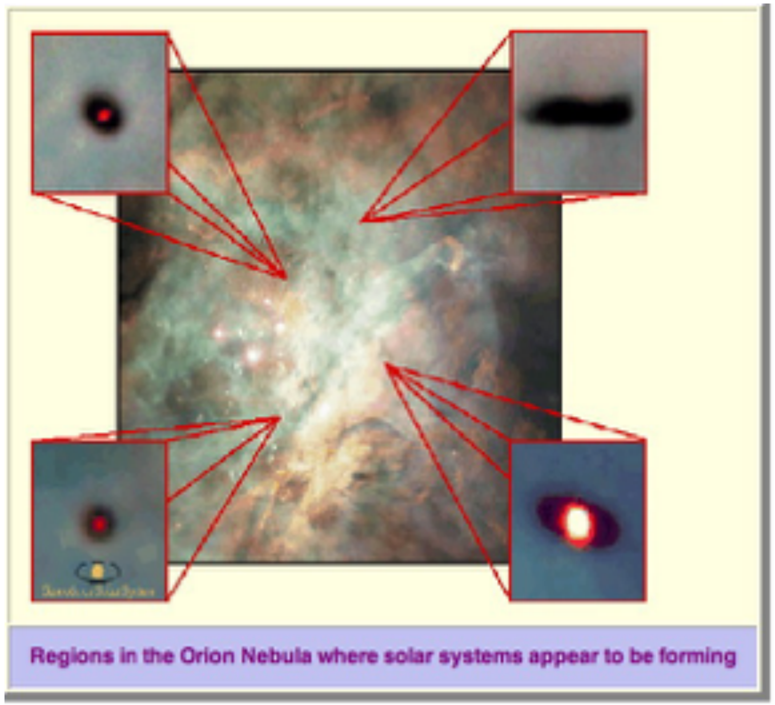
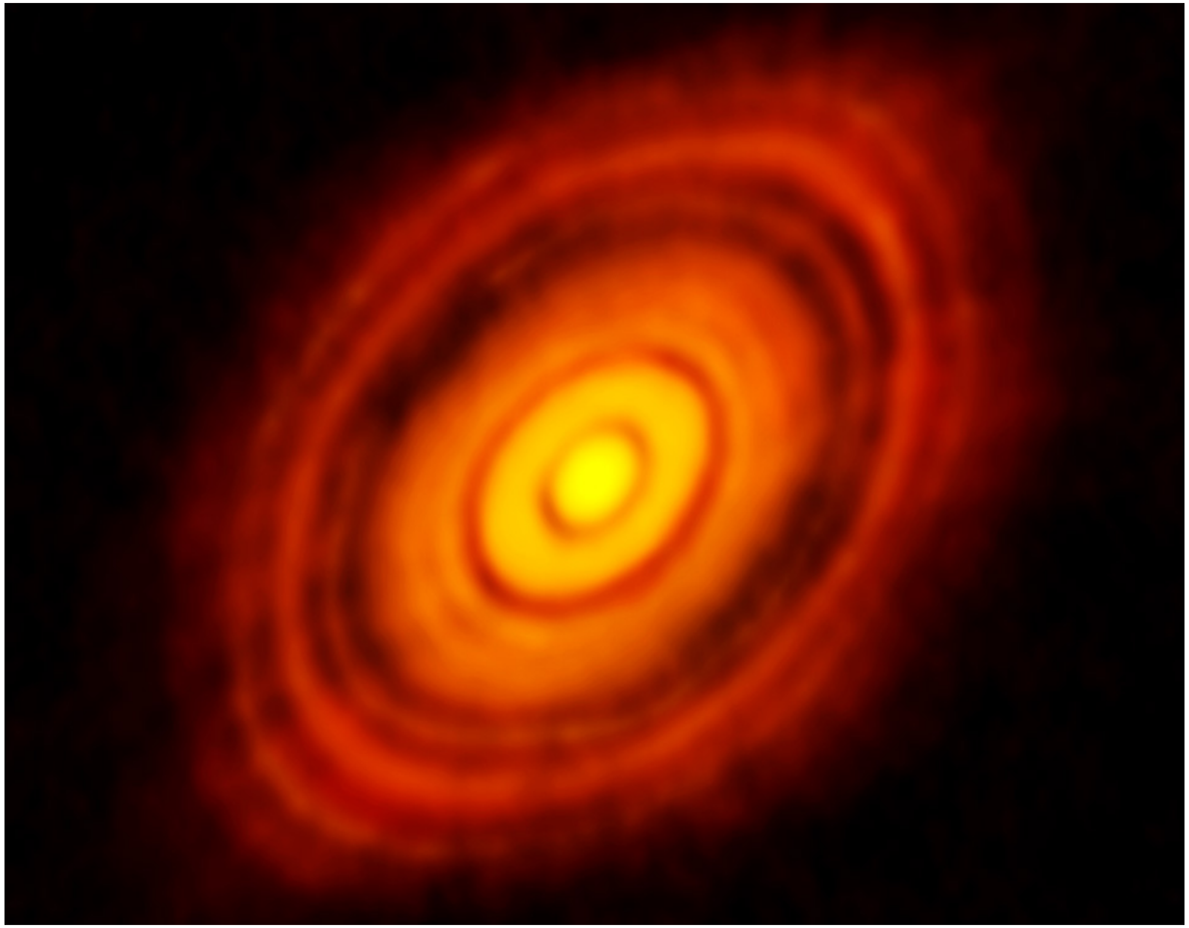
Lecture 4 – Protoplanetary disk structure

- Protoplanetary disk
  - Pressure distribution
  - Density distribution
  - Temperature distribution
- Cloud collapse & Disks



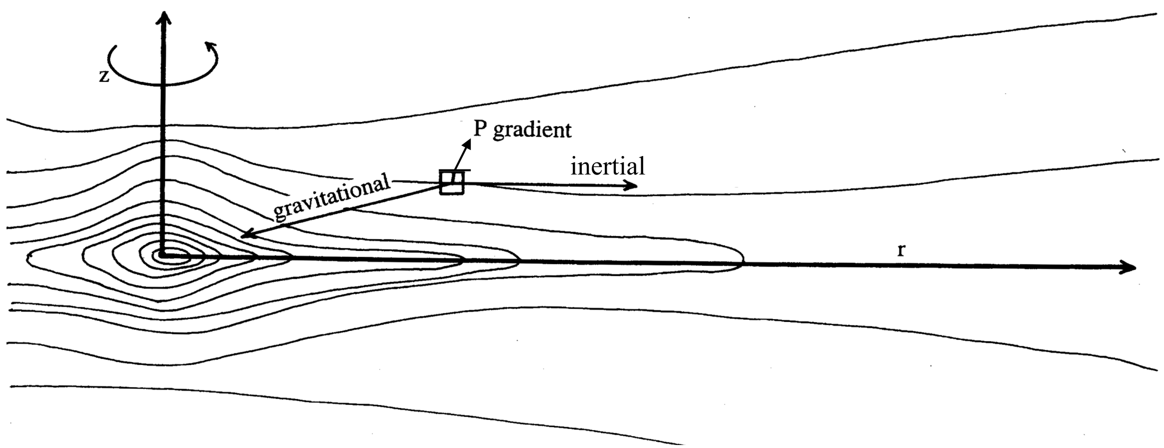
The Protoplanetary Disk of HL Tauri from ALMA



Gas pressure distribution

- Know surface mass and density of the early nebula ($\sigma\ (r) = \sigma_0\ r^{-3/2}$), but how is gas pressure, density and temperature distributed?

- Nebula can be considered a flattened disk, each element subject to three forces:
    1. Gravitational, directed inward.
    2. Inertial, directed outward.
    3. Gas pressure, directed upward and outward.



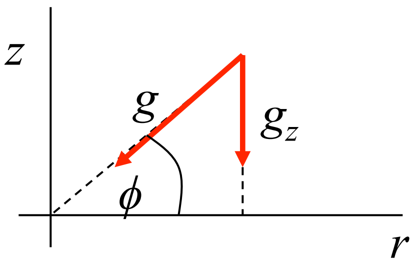
Gas pressure distribution

- Assume pressure gradient balances z-component of gravity. Hydrostatic equilibrium therefore applies:

$$dP = -\rho g_z dz \qquad \textit{Eqn. 1}$$

- But, $g_z = g \sin\phi$

$$= \left[\frac{GM}{(r^2 + z^2)}\right]\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$$



- As $(r^2 + z^2) \sim r^2 \Rightarrow$ $g_z = \frac{GMz}{r^3}$ *Eqn. 2*

- Substituting *Eqn. 2* and Perfect Gas Law ($P = \rho RT/\mu$) into *Eqn. 1:*

$$dP = -\left(\frac{P\mu}{RT}\right)\left(\frac{GMz}{r^3}\right)dz$$

Gas pressure distribution

- Rearrange: $$\int_{P_c}^{P_z} \frac{1}{P} dP = -\left(\frac{\mu GM}{RTr^3}\right) \int_0^z z dz$$

  where P_c is the pressure in the central plane, and P_z is the pressure at a height z.

- Integrating, $$P_z = P_c \exp\left(-\frac{\mu GMz^2}{2RTr^3}\right)$$ *Eqn. 3*

- The isothermal pressure distribution is flat for small z, but drops off rapidly for larger z.

Gas pressure distribution

- P_z drops to 0.5 central plane value, when

$$P_c/2 = P_c \exp\left(-\frac{\mu G M z^2}{2RTr^3}\right)$$

- Rearranging, $\ln(0.5) = -0.7 = -\frac{\mu G M z^2}{2RTr^3}$

$$\Rightarrow z = \sqrt{\frac{1.4RTr^3}{\mu GM}}$$

- Using typical value => $z \sim 0.2\ AU$
(1 AU = 1.49 x 10^{13} cm)

- Thus z/r is extremely small for bulk of disk gas.

- Effective thickness of nebular disk as seen from the Sun is only a few degrees.

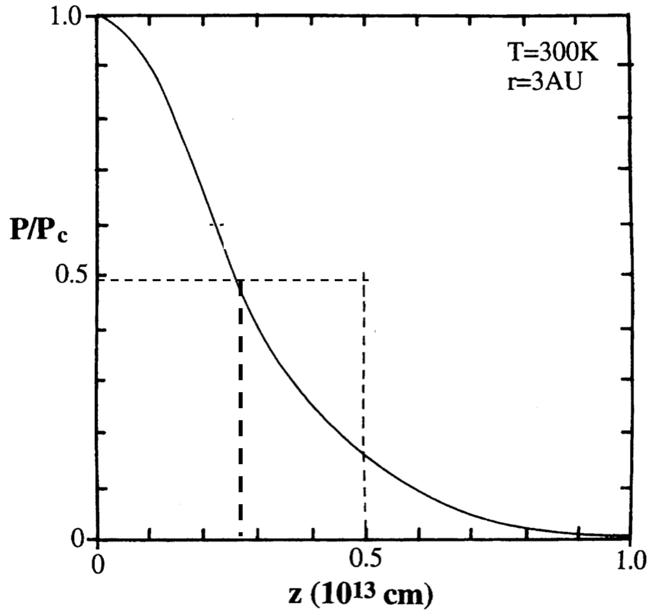


Figure from Lewis

Gas density distribution

- For a thin disk, $z << r$, and $g_z \cong \Omega^2 z$ where

$$\Omega = \sqrt{\frac{GM}{r^3}}$$

  is the Kelperian angular velocity.

- Using Eqn. 3, the vertical density has simple form:

$$\rho = \rho_0 e^{-z^2/2h^2}$$

- where ρ_0 is the mid-plane density $\rho_0 = \frac{1}{\sqrt{2\pi}} \frac{\sigma}{h}$

  and the vertical disk scale height is $h \equiv \frac{c_s}{\Omega} \Rightarrow h/r = M^{-1}$

Gas pressure distribution

- How does the central gas pressure vary with r? Know that the surface density is

$$\sigma = \int_{-\infty}^{+\infty} \rho dz$$
$$= \int_{-\infty}^{+\infty} P_z \mu / RT dz$$
$$= \frac{\mu P_c}{RT} \int_{-\infty}^{+\infty} \exp\left(-\frac{\mu GM}{2RTr^3}\right) z^2 dz$$

which is a standard definite integral

$$\sigma = (\mu P_c/RT)(2\pi RTr^3/\mu GM)^{1/2}$$

Will be given if examined

or

$$\sigma / P_c = (2\pi\mu r^3/RTGM)^{1/2} = 2160 r^{3/2} T^{1/2}$$

$$P_c = \sigma\, r^{-3/2} T^{-1/2}/2160$$

and using $\sigma = 3300\, r^{-3/2}$

$$\Rightarrow P_c = 1.5 T^{-1/2} r^{-3} \qquad \textit{Eqn. 4}$$

Gas temperature distribution

- In z-direction, temperature gradients are very large ($dT/dz >> 0$), due to the large temperature difference between the interior and the edge of the disk and the very thin disk in the vertical direction. This drives convection.

- We know that $P = \rho RT/\mu$ and $R \sim C_p$ for the solar nebula.

- Therefore, $\frac{dP}{dz} = \frac{\rho C_P}{\mu}\frac{dT}{dz}$

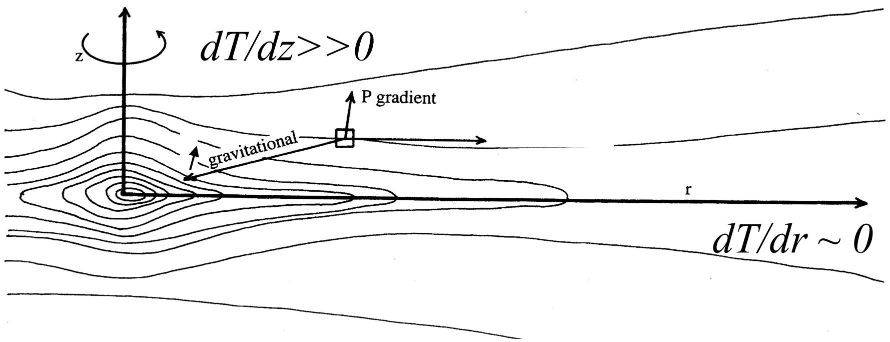
$$\Rightarrow \frac{dT}{dz} = \frac{\mu}{\rho C_p}\frac{dP}{dz}$$

- As $dP/dz = -\rho g$;

$$\frac{dT}{dz} = \frac{\mu}{\rho C_p}(-\rho g) = -\mu g / C_P$$

- And using $g = GMz / r^3$:

$$\frac{dT}{dz} = -\frac{\mu GMz}{C_P r^3} = -1.5 \times 10^8 z r^{-3}$$



Gas temperature distribution

- Horizontally (in r-direction), temperature gradient is not significant. As little heat enters or leaves the disk in this direction, the gas behaves adiabatically.

- The temperature and pressure changes can therefore be related by:

$$P/P_0 = (T/T_0)^{Cp/R}$$

- For $150 < T < 2000K$, $C_p/R \sim 7/2$ for the nebula, therefore:

$$P_c \sim T_c^{7/2} \text{ or } T_c \sim P_c^{2/7}$$

- From *Eqn. 4,* we know that $P_c \sim r^{-3}$, thus: $T_c \sim (r^{-3})^{2/7} \sim r^{-6/7}$

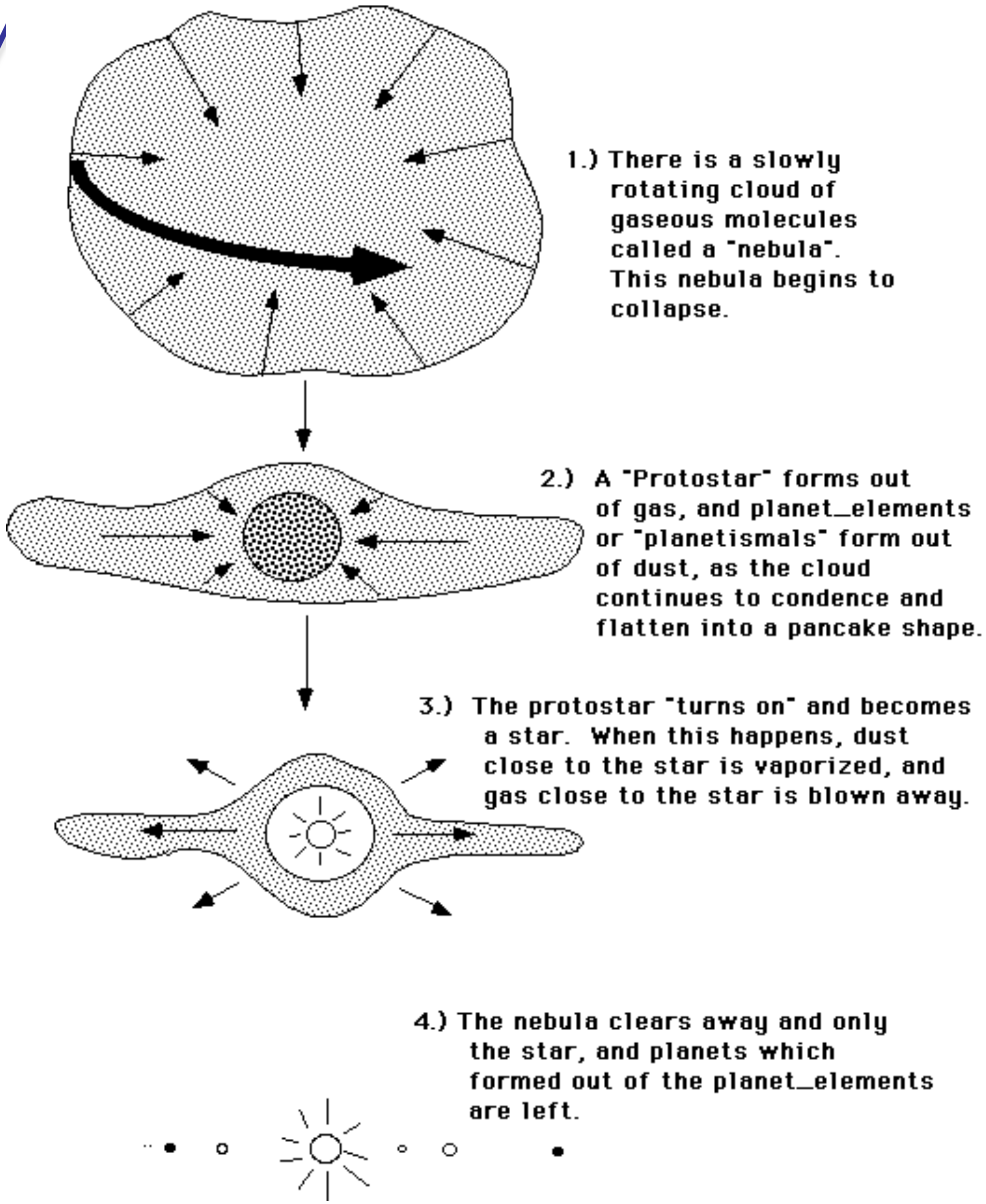
- The temperature therefore falls off relatively slowly with r.

Gas properties in protoplanetary disk

- Surface density: $\sigma \sim r^{-3/2}$
- Force due to gravity: $g(z) = GMz / r^3$
- Pressure in *z*: $P_z = P_c \exp\left(-\frac{\mu GMz^2}{2RTr^3}\right)$
- Pressure with *r*: $P_c = 1.5T^{-1/2}r^{-3}$
- Density with r $\rho = \rho_0 e^{-z^2/2h^2}$
- Temperature with *z*: $dT \sim -\frac{\mu GMz}{C_P r^3} dz$
- Temperature with *r:* $T_c \sim r^{-6/7}$
- See Chapter *IV* of *Physics and Chemistry of the Solar System* by Lewis.

Cloud Collapse and Star/ Planet Formation

- Jeans cloud collapse equations describe the conditions required for an ISM cloud to collapse.
- As a cloud collapses, central temperature increases.
- This is accompanied by spinning-up of the central star (to conserve AM).
- Disk also flattens into an oblate spheroid.



Stars and disks in Yong Stellar Objects (YSOs)

- Class 0: Gravity causes cloud to fragment into dense cores. One core further collapses, becoming hot enough to start emitting in far IR.

- Class I: By Cloud will start to rotate faster and dust will settle in disk. Inner part of cloud will start to fuse hydrogen => star is born. Spectrum does not look like a star yet as is embedded in envelope of gas and dust.

- Class II: Envelope dissipated due to stellar wind. Due to the dense disk, object will still radiate in the IR.

- Class III: Consist of young star surrounded by a transparent disk. Disk matter is discarded through various mechanisms such as photo-evaporation and accretion onto central star.

- Planet formation thought to occur during transition from class II to class III.