Lecture 5: Planetary system formation theories
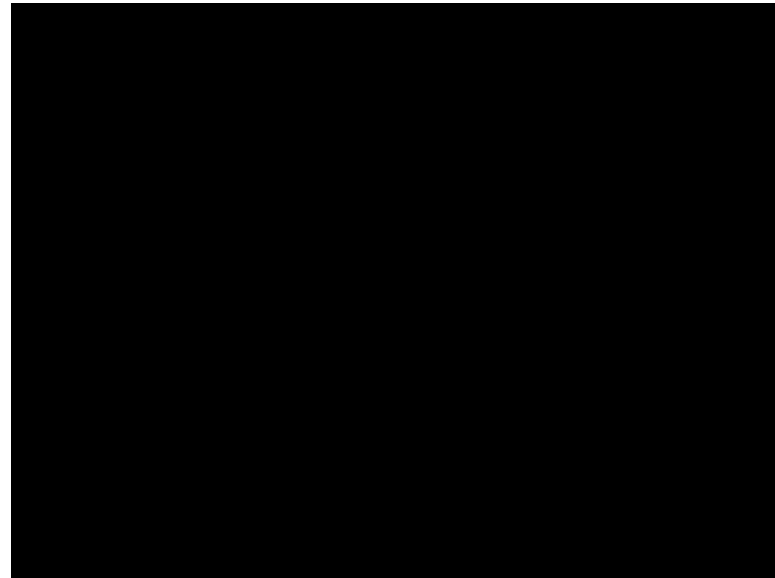
o Topics to be covered:

    o Laplace nebula theory

    o Jeans' tidal theory

    o Capture theory

Planet formation models

o Three basic models have been proposed:

  o **Tidal theory**: Planets formed from condensed gasses 'ripped' from an all ready formed Sun.

  o **Capture theory**: During a close stellar encounter, Sun captures material out of which planets form.

  o **Nebula theory**: Planets formed at the same time as the Sun in the same gas cloud.

Laplace nebula theory

o   Proposed by Pierre Laplace in *Exposition du Système du Monde* (1796).

o   Consists of 5 stages:

1. Slowly rotating, collapsing gas and dust sphere.
2. An oblate spheroid, flattened along the spin axis.
3. The critical lenticular form - material in equatorial region is in free orbit.
4. Rings left behind in equatorial plane due to further collapse. "Spasmodic" process leads to annular rings.
5. One planet condenses in each ring with Sun at centre.
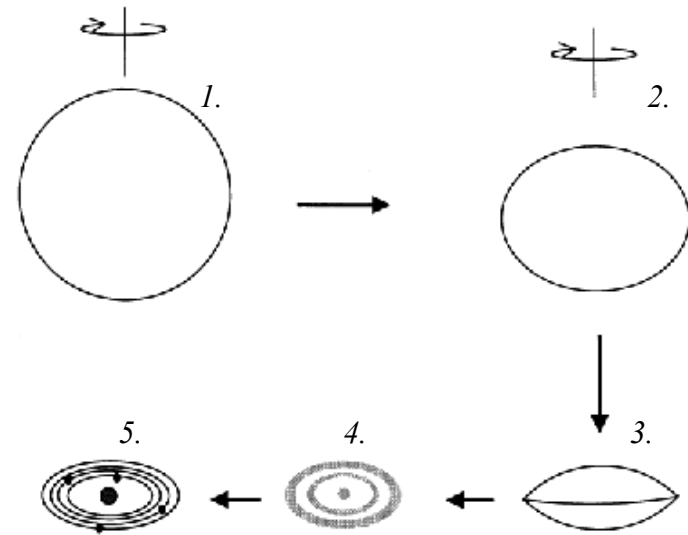
Laplace nebula theory: Difficulties

1. Strongest criticism related to the distribution of AM.

    o   There is no mechanism for the partitioning of mass and AM.

    o   Mass and AM concentrated on the central star.

2. While still a student at Cambridge, Maxwell suggested that differential rotation between inner and outer parts of rings would prevent material from condensing.

    o   Gravitational attraction between objects in the rings would not be sufficient to overcome inertial forces.

   o   Rings would require much more mass than the planets they formed to overcome this effect.

Jeans' tidal theory

o Laplace theory was a *monistic* theory - same body of material in a single process gave rise to both the Sun and the planets.

o James Jeans (1917) proposed a *dualistic* theory that separated formation of Sun from formation of planets.

o Jeans' Theory involved interaction between Sun and a very massive star in three stages:

1. Massive star passes within Roche Limit of Sun, pulling out material in the form of a filament.
2. Filament is gravitationally unstable, and breaks into series of blobs of masses greater than the Jeans' critical mass, and so collapse to form proto-planets.
3. Planets were left in orbit about the Sun.
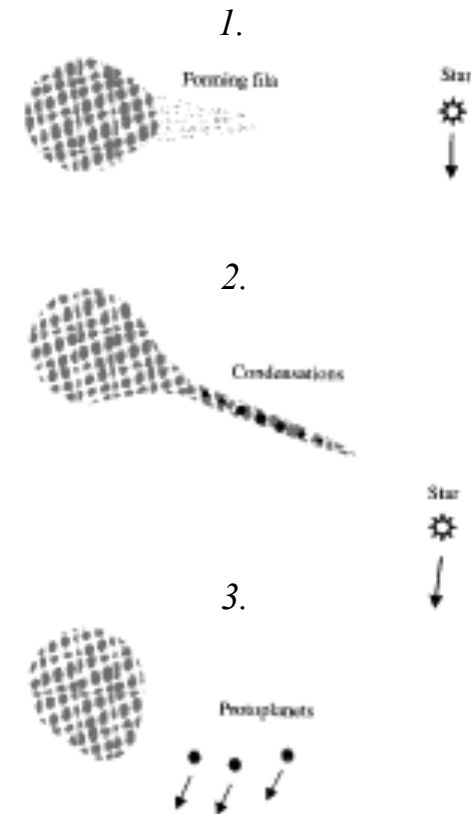
Roche Limit

o   Roche limit is distance at which a satellite begins to be tidally torn apart.

o   Consider M with 2 satellites of mass m and radius r orbiting at distance R. Roche limit is reached when m is more attracted to M then to m. Occurs when $F_{tidal} \geq F_{binding}$
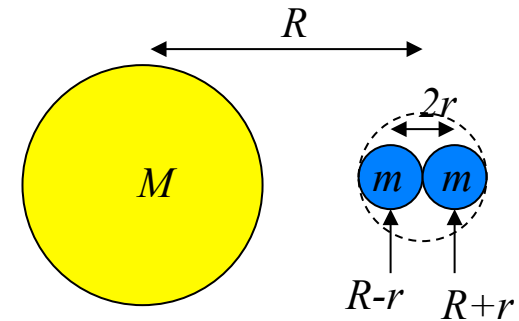
o   The binding force is: $\boxed{F_{binding} = \dfrac{Gm^2}{(2r)^2}}$ *Eqn. 4*

o   Force of attraction between mass M and nearer satellite is: $GMm/(R - r)^2$. Force on more distant satellite is $GMm/(R + r)^2$

o   Tidal force experienced is thus, $\quad F_{tidal} = GMm\left(\dfrac{1}{(R-r)^2} - \dfrac{1}{(R+r)^2}\right)$

$$= \dfrac{GMm}{R^2}\left((1 - r/R)^{-2} - (1 + r/R)^{-2}\right)$$

$$= \dfrac{GMm}{R^2}\left((1 + 2r/R) - (1 - 2r/R)\right) \Longrightarrow \boxed{F_{tidal} = \dfrac{4GMmr}{R^3}}\quad \textit{Eqn. 5}$$

Roche Limit (cont.)

o We can therefore rewrite the inequality $F_{tidal} \geq F_{binding}$ as: $\dfrac{4GMmr}{R^3} \geq \dfrac{Gm^2}{4r^2}$

o Rearranging then gives $R_{Roche} = \left(\dfrac{16Mr^3}{m}\right)^{1/3}$

o As $M = 4/3\ \pi R^3 \rho_M$ and $m = 4/3\ \pi r^3 \rho_m$:

$$R_{Roche} = 2.520R\left(\dfrac{\rho_M}{\rho_m}\right)^{1/3}$$ *Approximate Roche Limit*

o Objects which pass within this are torn apart.

o The Earth's Roche limit is ~18,470 *km*.

Roche Limit Numerical Simulation

o Example of a quasi-statically collapsing protostar encounter, with parameters equal to that of Woolfson's 1964 paper.

o The circle represents the Roche limit of the protostar, calculated from the central density of the protostar.

o At $t = 250$ years the Roche limit of the material in the filament is around .
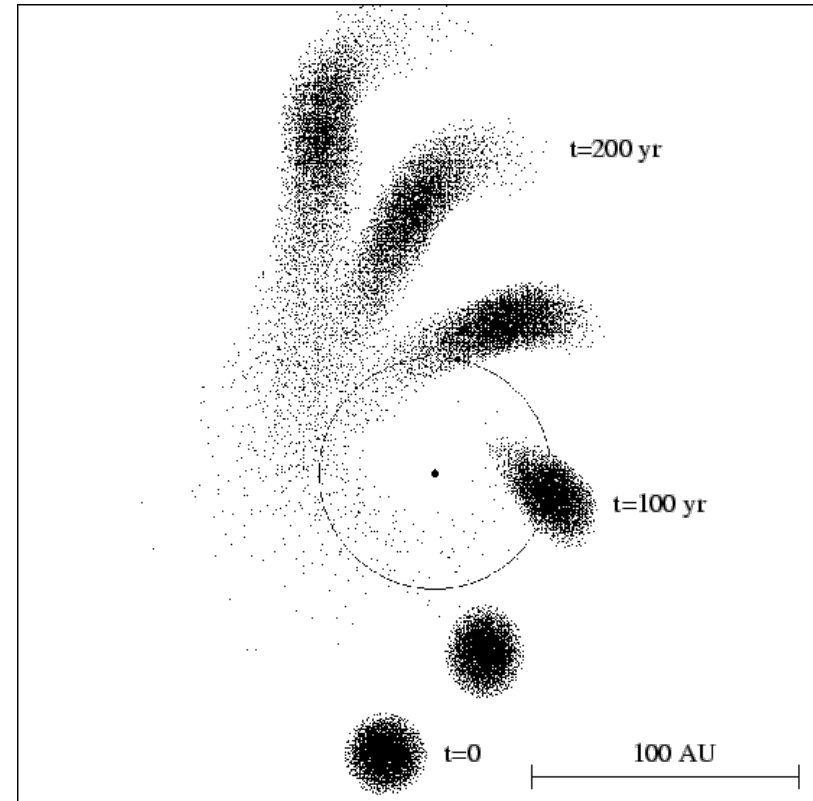
Jeans' tidal theory: Proto-planet formation

o Jeans showed that filament would be unstable, and break into series of proto-planets.

1. Small density excess at *A*.

2. Gravitational attraction causes material in *B* and *B'* to move towards *A*.

3. Material at *C* and *C'* now experience an outward force and produce high-density regions at *D* and *D'*.

o Jeans treated as a wave-like problem, finding average distance between proto-planets to be:

$$\lambda = c_s t_{ff} = \frac{c_s}{(G\rho)^{1/2}}$$

where $c_s \sim (kT/m)^{1/2}$ is the sound speed. Called the Jeans length (compare to Jeans' radius).
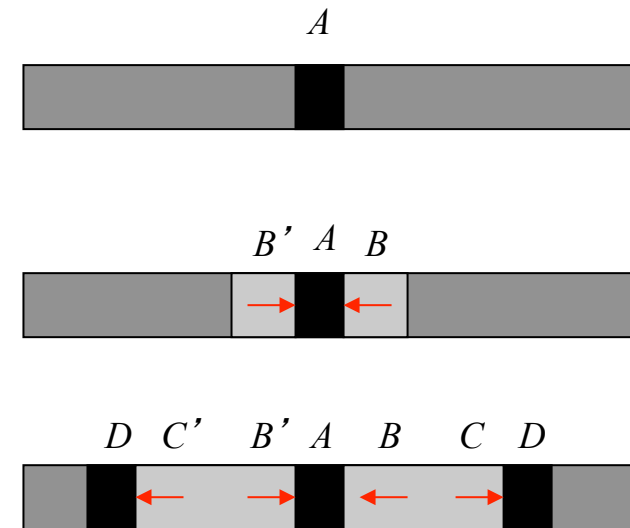
Jeans Length Derivation

o Consider filament of gas in 1D. Evolution is governed by:

o Mass continuity:
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \qquad (1)$$

o Equation of motion:
$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u u) = -\frac{\partial P}{\partial x} - \rho \frac{\partial \Phi}{\partial x} \qquad (2)$$

o Gravitational potential:
$$\frac{\partial^2 \Phi}{\partial x^2} = 4\pi G \rho \qquad (3)$$

o If gas is isothermal,
$$P = c_s^{\,2} \rho \qquad (4)$$

Jeans Length Derivation

o   Assume gas initially has constant density (ρ_0) and pressure (P_0) and is at rest ($u_0=0$) in a potential (Φ_0).

o   Now consider a small perturbation, such that

$$\rho = \rho_0 + \rho_1 \qquad P = P_0 + P_1 \qquad u = u_0 \qquad \Phi = \Phi_0 + \Phi_1$$

$$(5)$$

o   Also assume perturbation is isothermal, so $P_1 = c_s^2 \rho_1$

Jeans Length Derivation

o Inserting relations from (5) into Eqns. 1, 2 and 3 and linearising:

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial u_1}{\partial x} = 0$$

$$\frac{\partial u_1}{\partial t} = -\frac{\partial \Phi_1}{\partial x} - \frac{c_s^2}{\rho_0} \frac{\partial \rho_1}{\partial x}$$

$$\frac{\partial^2 \Phi_1}{\partial x^2} = 4\pi G \rho_1$$

o If assume solution of form exp[$i(kx-\omega t)$], therefore $\dfrac{\partial}{\partial x} = ik$ and $\dfrac{\partial}{\partial t} = i\omega$

$$\omega \rho_1 + k\rho_0 u_1 = 0$$

$$\frac{kc_s^2}{\rho_0}\rho_1 + \omega u_1 + k\Phi_1 = 0$$

$$4\pi G \rho_1 + k^2 \Phi_1 = 0$$
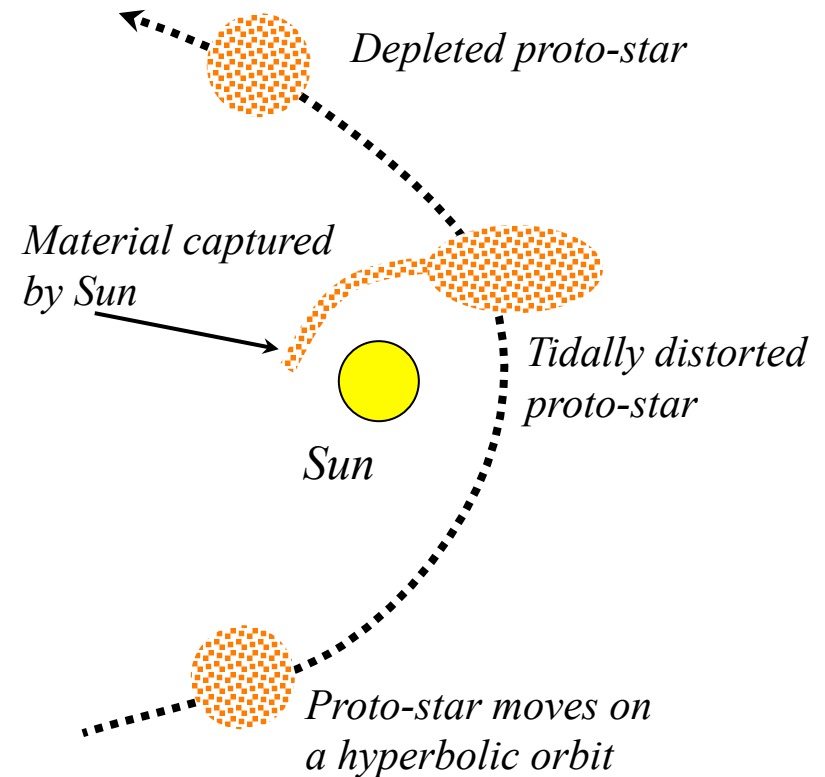
Jeans Length Derivation

o This set of linear equations has a solution if

$$
\begin{vmatrix}
\omega & k\rho_0 & 0 \\
\dfrac{kc_s^2}{\rho} & \omega & k \\
4\pi G & 0 & k^2
\end{vmatrix} = 0
$$

o i.e., if $\quad \omega^2 = k^2 c_s^2 - 4\pi G\rho_0$

o If k is large, $k^2 c_s^2 - 4\pi G\rho_0 > 0 \Rightarrow \omega$ is real and perturbation is stable.

o If $k^2 c_s^2 - 4\pi G\rho_0 < 0 \Rightarrow \omega$ is imaginary and perturbation grows exponentially.

o Border between regimes is a critical wavenumber $\quad k_J = \left(\dfrac{4\pi G\rho_0}{c_s^2} \right)^{1/2}$

or to a *critical wavelength* $\quad \lambda_J = c_s \left(\dfrac{\pi}{G\rho_0} \right)^{1/2}$

o Therefore, a perturbation with $\lambda > \lambda_J$ will collapse.

Jeans' tidal theory: Difficulties

1. Very massive stars are rare and distant.
   o Probability of massive star coming close to another star is therefore very low.
   o Sun's nearest companion is Proxima Centauri ($d = 1.3\ pc \Rightarrow R_{sun}/d \sim 2\times10^{-8}$).

2. Rotational period of Sun and Jupiter should be similar if Jupiter's material was from Sun.
   o Not the case ($P_{sun} \sim 26$ days and $P_{jupiter} \sim 10$ hours).

3. In 1935, Henry Russell argued that it is not possible for the material from the Sun to acquire enough AM to explain Mercury, let alone the other planets.

4. Spitzer (1939) noted that material with solar densities and temperatures would give a minimum mass for collapse of ~100 times that of Jupiter.
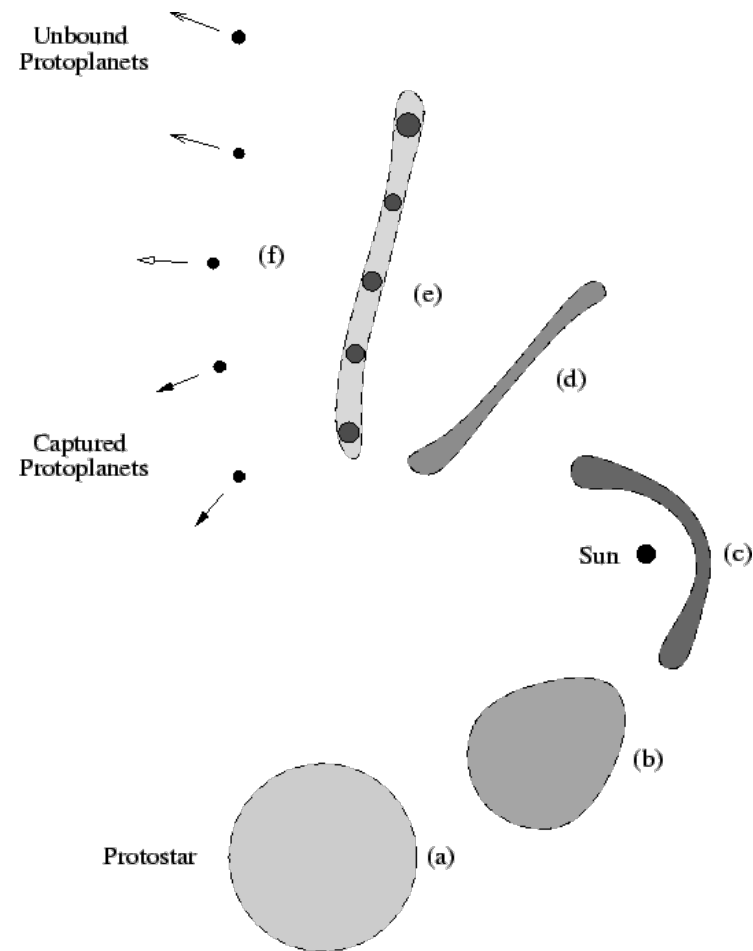
Capture theory

o Modified version of Jeans' theory, proposed in 1964 by M. Woolfson.

o Sun interacts with nearby protostar, dragging filament from protostar.

o Low rotation speed of Sun is explained as due to formation before planets.

o Terrestrial planets due to collisions between protoplanets close to Sun and giant planets.

o Planetary satellites due to condensation in drawn out filaments.

Depleted proto-star

Material captured by Sun

Tidally distorted proto-star

Sun

Proto-star moves on a hyperbolic orbit

Tidal induced fragmentation followed by capture

o   (a) The protostar approaches on a hyperbolic orbit, (b) As it collapses, it deforms into an egg shape, (c) The whole protostar is stretched into an arc shaped filament of material at perihelion, (d) As the filament leaves perihelion it straightens up, (e) The filament fragments to produce several protoplanetary condensations, (f) In parabolic encounters, upto half of the protoplanets are captured into high eccentricity orbits.

Capture theory: Difficulties

o Space between local stars too large for 9 planets and 60 moons to be caught by Sun.

    o Millions would have to pass, in order for one to be caught.

o Planets would tend to spiral into Sun, not begin encircling it.

o Moons would not begin orbiting around planets; they would crash into Sun or into planets.

o Cannot explain why Sun and Planets have the same apparent age (4.5 Gyrs).