Lectures 6-8: Solar nebula theory
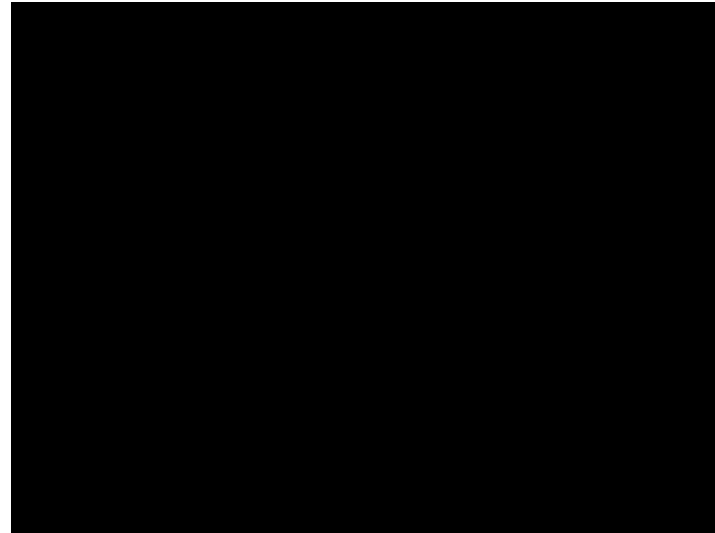
o   Topics to be covered:

   o   Solar nebula theory

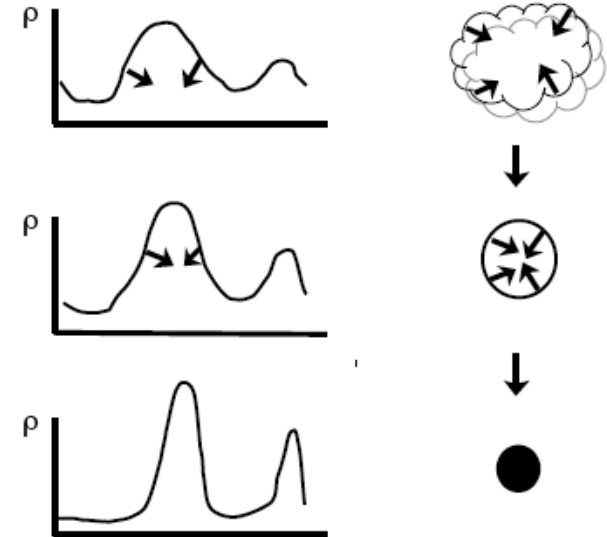# *Solar nebula theory*

o   Five steps to planet formation in Solar nebula theory.

1. Collapse
   o   Heating via conversion of PE to KE.
2. Spinning
   o   Spinning up of material to conserve AM.
3. Flattening
   o   Sphere to disk due to rotation.
4. Condensation
   o   Gas to liquid and solid particles due to cooling.
5. Accretion
   o   Solid particles 'stick' due to electrostatic and gravitational forces.

o   See extensive review by Lissauer:
         http://adsabs.harvard.edu/abs/1993ARA%26A..31..129L

1. Collapse - Solar Nebula Theory

o For a cloud with $M > M_J$ where M_J [?] $(T^3/$[?]$)^{1/2} =>$ cloud collapses.

o Localised density enhancement ([?][?]/[?]) may cause collapse.

o Gravity pulls material in, gas pressure opposes.

o When pressure $>>$ gravity $=>$ oscillations.

o When pressure $<<$ gravity $=>$ collapse.

o Cooling lowers pressure, can trigger collapse.

1. Collapse - Solar Nebula Theory

o   Consider N molecules of mass m in a volume of size L at a temperature T.

o   Gravitational energy: $E_G \sim -\dfrac{GMM}{L}$ where $M = Nm \sim L^3\boxed{?}$.

o   Thermal energy: $E_T \sim NkT$

o   Ratio is: $\dfrac{E_G}{E_T} \sim \dfrac{GM^2}{LNkT} \sim \dfrac{G(\rho L^3)m}{LkT}$

$$= \left(\frac{L}{L_J}\right)^2$$

where L_J is the *Jeans' length:*

$$\boxed{L_J \sim \left(\frac{kT}{G\rho m}\right)^{1/2}}$$

o   Gravity wins when $L > L_J$.

1. Collapse - Solar Nebula Theory

o   Collapse timescale can be also be estimated from gravitational acceleration:

$$g \sim \frac{GM}{L^2} \sim \frac{L}{t^2}$$

o   Time to collapse:

$$t \sim \left(\frac{L}{g}\right)^{1/2} \sim \left(\frac{L^3}{GM}\right)^{1/2}$$

$$\Rightarrow t_G \sim \left(\frac{1}{G\rho}\right)^{1/2}$$

=> denser regions collapse faster.

o   Ignoring gravity, pressure waves travel at sound speed: $c_s \sim \left(\frac{kT}{m}\right)^{1/2}$

o   Sound crossing times is therefore, $t_s \sim \frac{L}{c_s} \sim L\left(\frac{m}{kT}\right)^{1/2}$

 => Small hot regions oscillate more rapidly.
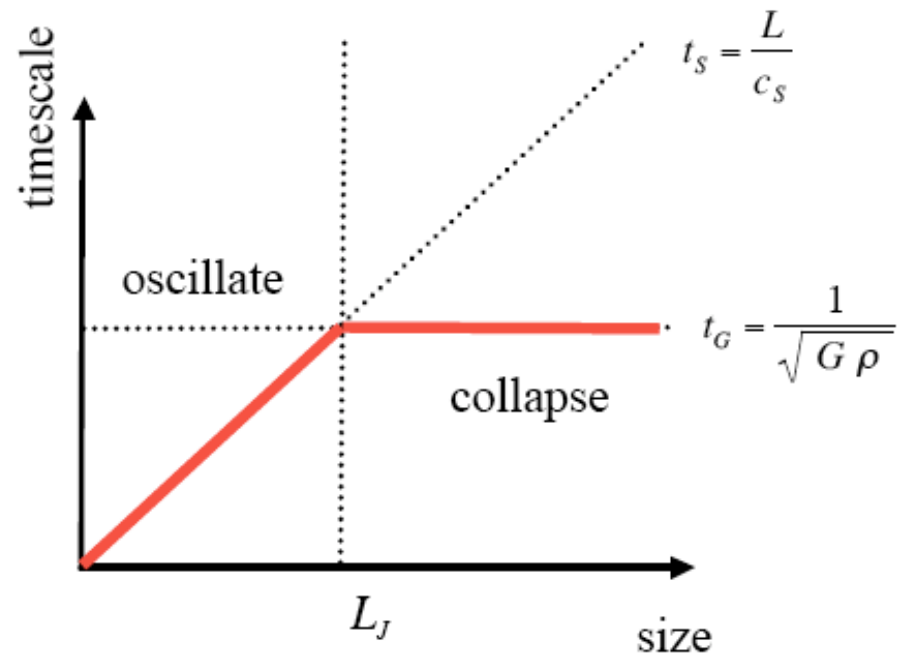
1. Collapse - Solar Nebula Theory

o Ratio of the collapse time (t_G) to the sound crossing time (t_s) is:

$$\frac{t_s}{t_G} \sim \frac{L\sqrt{G\rho}}{c_s} \sim L\left(\frac{G\rho m}{kT}\right)^{1/2}$$

$$\sim \frac{L}{L_J}$$

where L_J is again the *Jeans length:*

$$L_J \sim c_s\left(\frac{1}{G\rho}\right)^{1/2}$$

o Compare with form given in *Jeans formation theory.*

1. Collapse - Solar Nebula Theory

o   As gas cloud collapses, temperatures rise as potential energy converted to kinetic via:

$$E = KE + U = const$$

o   From Conservation of Energy, KE increases as U decreases.

o   Temperature therefore rises as $1/2mv^2 = 3/2\ kT$ or $T = 1/3k\ (1/2\ m\ v^2)$.

o   Some energy is radiated away thermally. The solar nebula becomes hottest near its center, where much of the mass collect to form the *protosun*.

o   Protosun eventually becomes so hot that nuclear fusion ignited in its core.
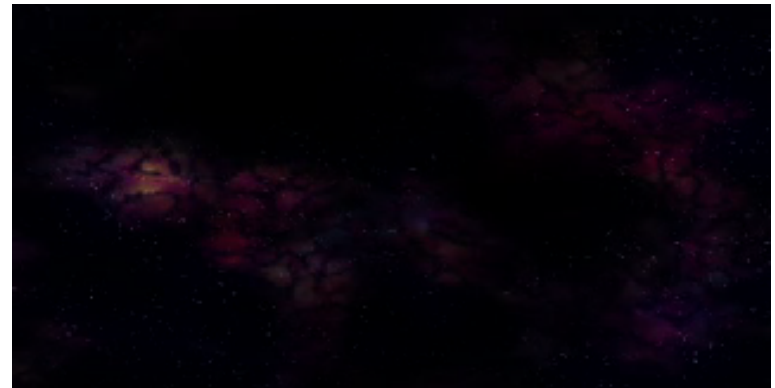
1. Collapse - Solar Nebular Theory

o   Initially, gas and dust with low AM fall to core of cloud. Material with high AM cannot due to centrifugal forces.

o   As gas and dust fall to equatorial plane, it collides with material falling in other direction => energy of this motion is dissipated as heat.

o   Consider a parcel of gas which falls from infinity to a circular orbit r. If half of gravitaional energy is converted to orbital kinetic energy via

$$\frac{GM_{protostar}}{2r} = \frac{v_c^2}{2}$$

the remainder is available for heat.

o   For $M_{protostar} = 1\ M_{sun}$, $v_c = 30$ km/s at 1 AU => $T \sim 7$ x 10^4 K. This temperature is never reached as the time-scale for radiaitve cooling << heating time.

2. Spinning - Solar Nebula Theory

o Solar nebula "spins-up" as it collapses to conserve AM.

$$L_f = L_i$$

$$m\, v_f\, r_f = m\, v_i\, r_i$$

$$m\, r_f^2\, [?]_f = m\, r_i^2\, [?]_i$$

$$\Rightarrow [?]_f = [?]_i\, (r_i\, /\, r_f)^2$$

o As $r_i > r_f \Rightarrow [?]_f \gg [?]_i$. Cloud spins up rapidly as it contract.

o Need a *breaking* mechanism.

o Rotation ensures not all of material collapses onto the protosun: the greater the AM of a rotating cloud, the more spread out it will be along its equator.
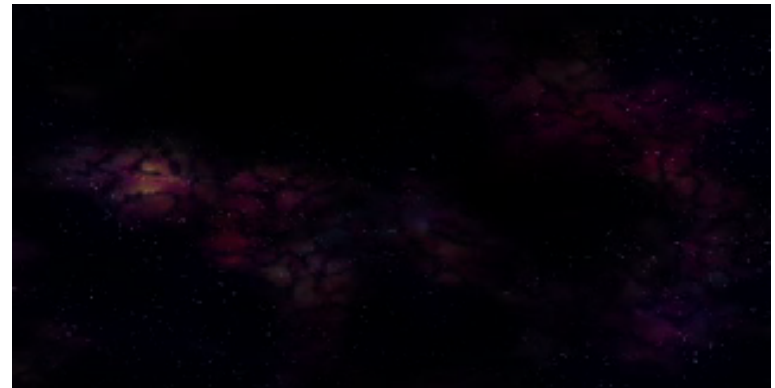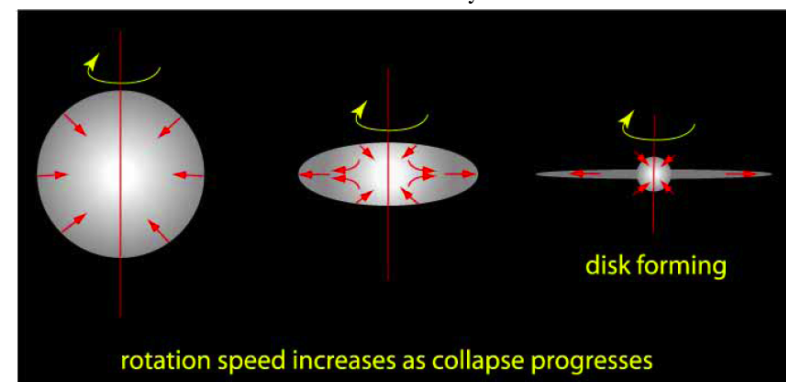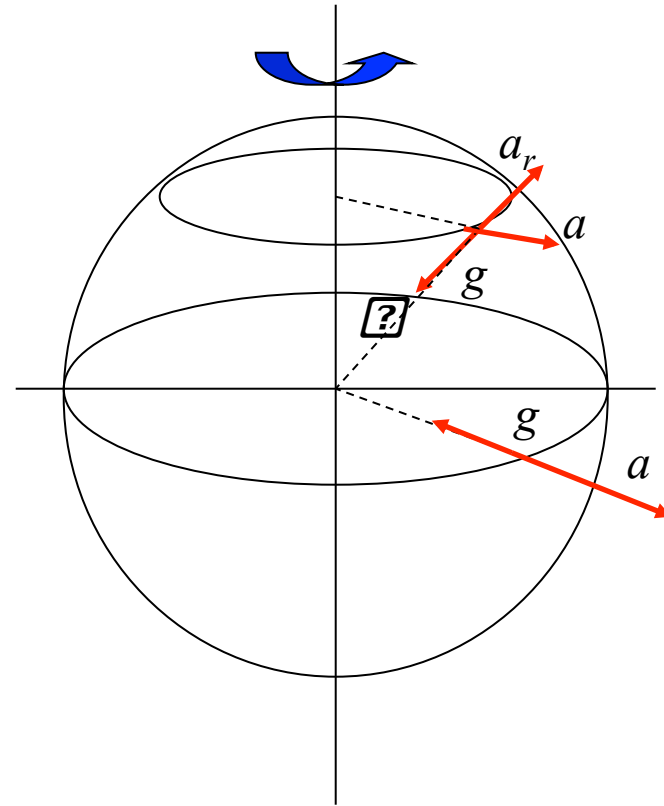
3. Flattening - Solar Nebula Theory

o $g = GM/r^2$ is directed *radially* to centre.

o $a = r \boxed{?}^2$ is *perpendicular* to rotation axis.
   Radial component is $a_r = r \boxed{?}^2 \sin \boxed{?}$.
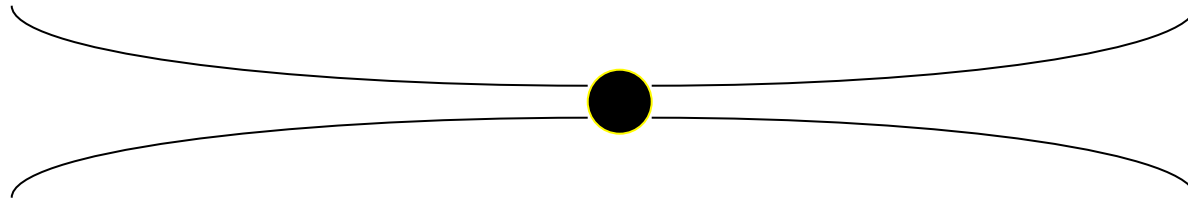
o   Net radial acceleration is

$$a(r) = g + a_r$$

$$a(r) = \frac{GM}{r^2} - r\omega^2 \sin\phi$$

o   At pole ($\boxed{?} = 0$) => $a(r) = GM/r^2$

o   At equator ($\boxed{?} = 90$) => $a(r) = g - r\boxed{?}^2$

o   In disk, there's a distance where $g = r\boxed{?}^2$ =>
   this is point where contraction stops.

disk forming

rotation speed increases as collapse progresses

3. Flattening - Properties of the disk

o   Basic properties of disk depend on how gas behaves in a gravity field.

    o   How does the disk shape determine?

    o   What is the velocity of disk?

    o   What is the density distribution of disk?

    o   How does rotation and gas pressure effect shape?

3. Flattening - Properties of the disk

o   In hydrostatic equilibrium

$$\frac{dP}{dz} = -\rho g$$

o z-component of gravity is

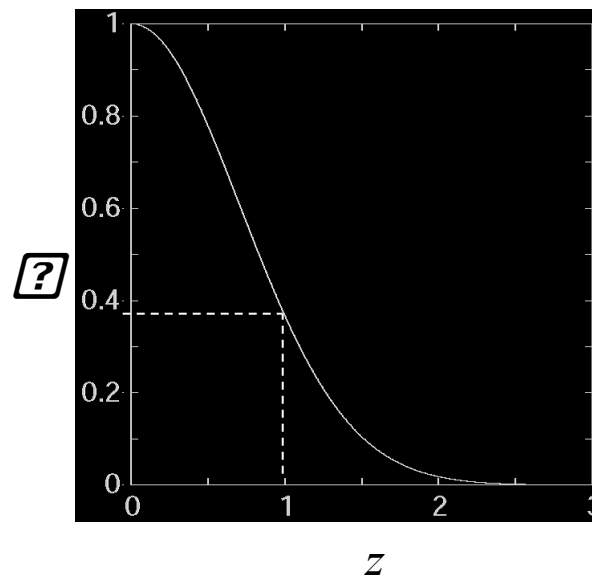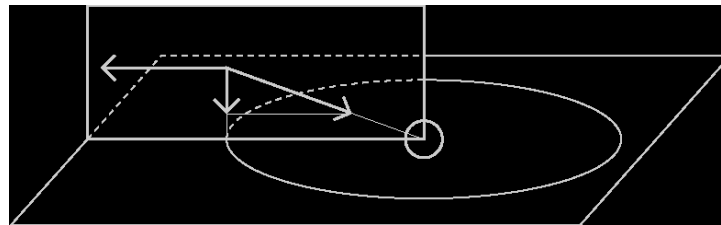$$g = -\omega^2 z$$

o   Equation of state for gas is

$$P = c_s^2 \rho$$

o   Density profile is thus:

$$\rho = \rho_0 \exp(-z^2 / H^2)$$

where the scale-height (ie thickness) is:

$$H = \sqrt{2} c \omega^{-1}$$

3. Flattening - Properties of the disk

o Disk aspect ratio is defined as: $\dfrac{H}{r} = \sqrt{2}\,\dfrac{c}{r\omega} \propto r^{(1-q)/2}$

o Condition for disk flaring is: *(1 - q) / 2 > 0 => q < 1*
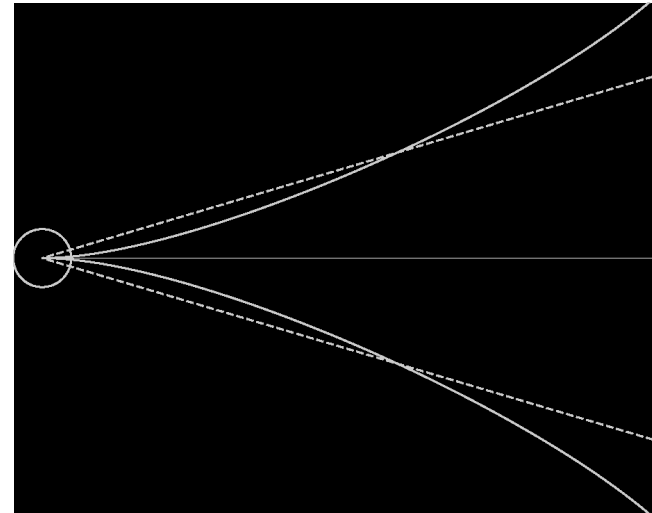
o For typical disks,

$$\frac{H}{r} \propto r^{1/4}$$

(ie *q = 1/2*)

o In general cases (eg. Galaxies)

$$\omega \propto r^{\beta} \qquad \Rightarrow \frac{H}{r} \propto r^{-q/2-\beta-1}$$

o Note, disk shape only depends on temperature, not density.

3. Flattening - Properties of the disk

o Radial force balance $F_{gravity} + F_{gas} + F_{rotation} = 0$

$$-\frac{GM_{star}}{r^2} - \frac{1}{\sigma}\frac{dP}{dr} + r\omega^2 = 0$$

where $\sigma = \sigma\,(r)$ is the surface density distribution.

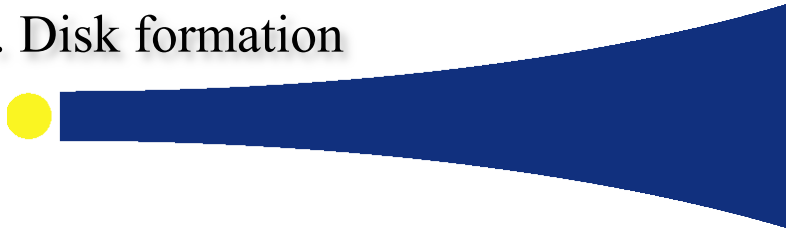o Angular velocity of gas is not strictly Keplarian:

$$\omega = \omega\,(1 - \eta)$$

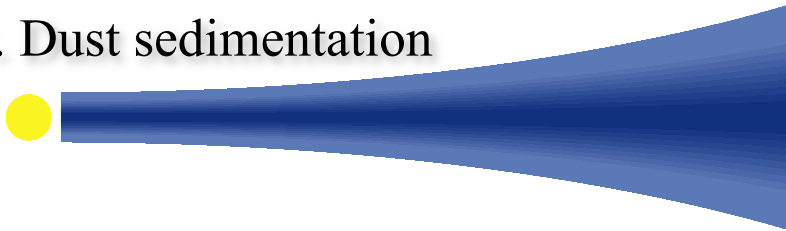where η is the viscosity $(0 < \eta < 1)$.

o Typically, $\eta \sim 0.001\text{-}0.01 \Rightarrow$ disk rotates slightly slower that would expect from Kepler's laws.

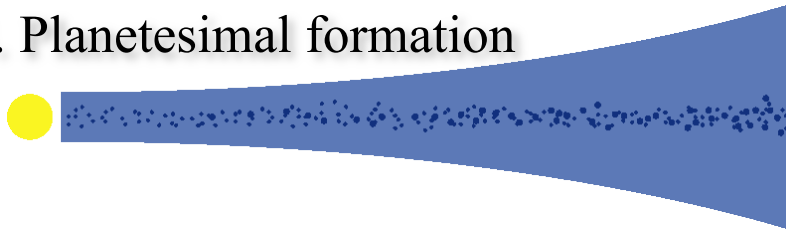# Summary of planet formation in disk
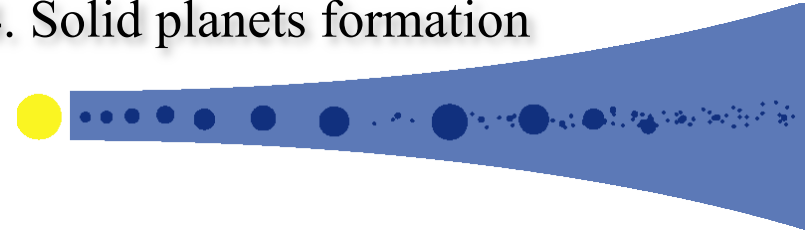
1. Disk formation
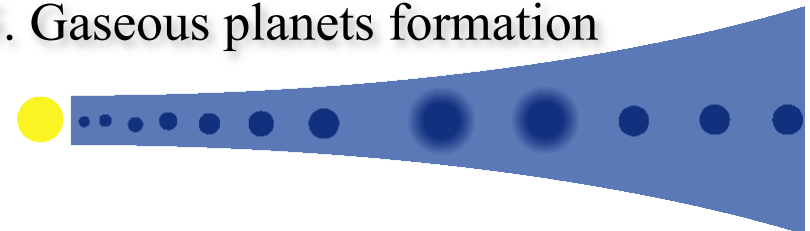
2. Dust sedimentation

3. Planetesimal formation

4. Solid planets formation

5. Gaseous planets formation

6. Disk dissipation

Step 4: Condensation

o Formation of planets requires "seeds" - chunks of matter that gravity can eventually draw together. Understanding these seeds and clumping is key to explaining the differing compositions of planets.

o The process by which seeds were sown is *condensation,* when solid or liquid particles condense out of a gas.

o Condensation is temperature dependent. When the temperature is low enough atoms/molecules solidify.
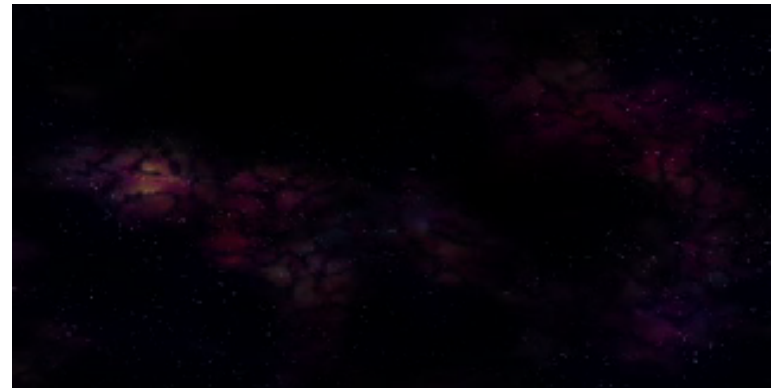
Step 4: Condensation

o   Approximate equation for the temperature variation in Solar Nebula is

$$T(r) = 631 / r^{0.77}$$

where *r* is in *AU*. "Ice line" where $T = 273\ K$ is located at ~3 *AU* from Sun.

o $T < 2,000\ K$, compounds of silicates (rock) and nickel-iron form.
o $T < 270$ K, carbon compounds, silicates and
ices form.

o   Planetary interiors to Mars
    o   Nebula temperature > 400 K
    o   Made of silicates and metals

o   Planets beyond Mars
    o   Nebula temperature < 300 K
    o   Made of silicates and ices
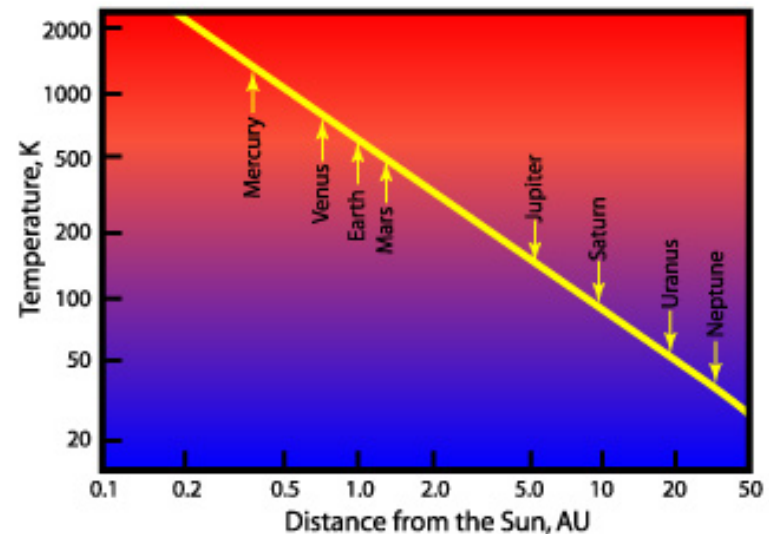
Step 4: Condensation

o   *Metals* include iron, nickel, aluminum. Most metals condense into solid at temperatures of 1000-1600 K. Metals made up <0.2% of the solar nebula's mass.

o   *Rocks* are common on Earth's surface, primarily silicon-based minerals (silicates). Rocks are solid at temperatures and pressures on Earth but melt or vaporize at temperatures of 500-1300 K depending on type. Rocky materials made up ~0.4% of the nebula by mass.

o   *Hydrogen compounds* are molecules such as methane (CH_4), ammonia (NH_3), and water (H_2O) that solidify into ices below about 150 K. These were significantly more abundant than rocks and metals, making up *~1.4%* of nebula's mass.

o   *Light gases (H* and *He)* never condense under solar nebula conditions. These gases made up the remaining 98% of the nebula's mass.

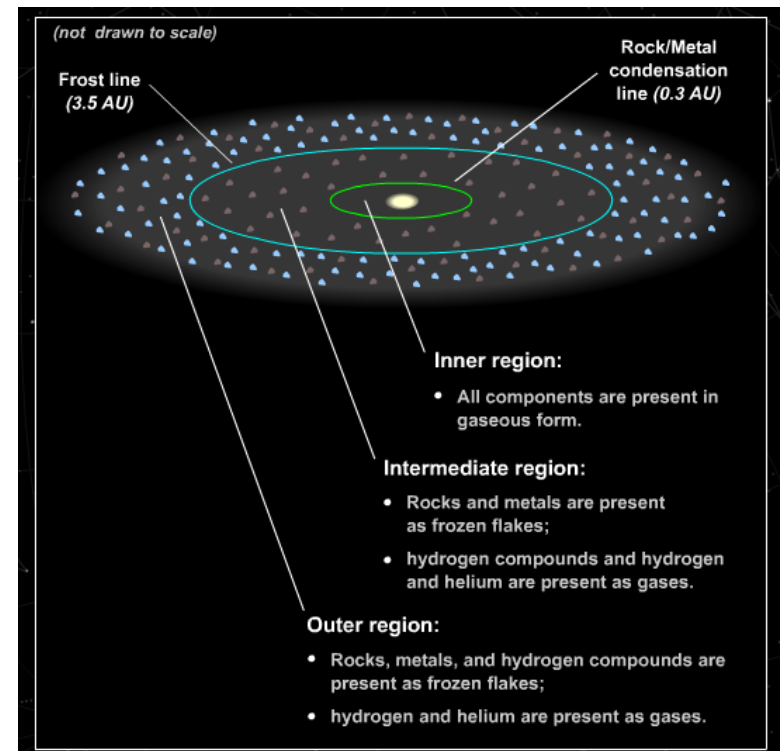o   Note: Order of condensation scales with density.

Step 4: Condensation

Materials in the Solar Nebula				
	Metals	**Rocks**	**Hydrogen Compounds**	**Light Gases**
Examples	iron, nickel, aluminum	silicates	water (H_2O) methane (CH_4) ammonia (NH_3)	hydrogen, helium
Typical Condensation Temperature	1,000–1,600 K	500–1,300 K	<150 K	(do not condense in nebula)
Relative Abundance (by mass)	·	▪	▪	■
	(0.2%)	(0.4%)	(1.4%)	(98%)

o   Terrestrial planets are made from materials that constituted ~0.6% of the nebula.

o   Jovian planets were formed in region where ~2% of material condensed. They also captured gas (98%).
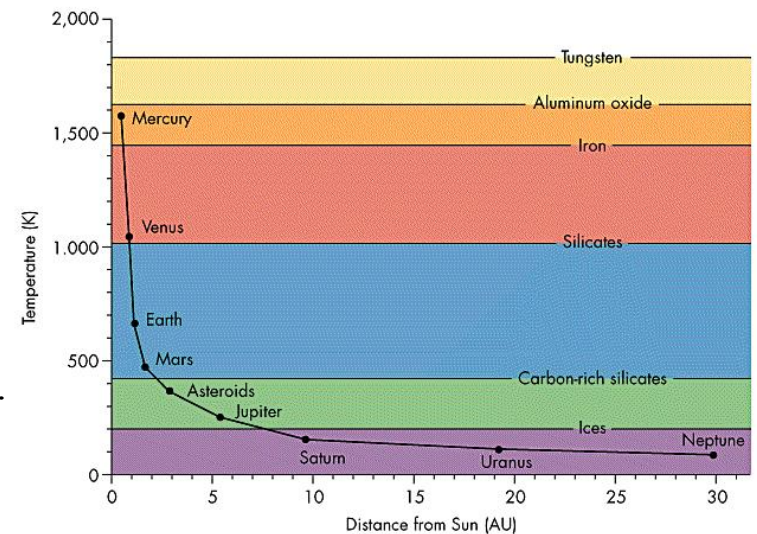
Step 4: Condensation

o *T~1500-2000K* at the present-day orbit of Mercury

   o About Mercury metals can begin to aggregate together

o Further out, rocky materials condense.

o Most metals/rocks condensing around the present-day orbit of Mars (*T~500K*).

o Hence inner planets have high metal/rock content and few volatile materials.

(not drawn to scale)

Frost line (3.5 AU)

Rock/Metal condensation line (0.3 AU)

Inner region:
- All components are present in gaseous form.

Intermediate region:
- Rocks and metals are present as frozen flakes;
- hydrogen compounds and hydrogen and helium are present as gases.

Outer region:
- Rocks, metals, and hydrogen compounds are present as frozen flakes;
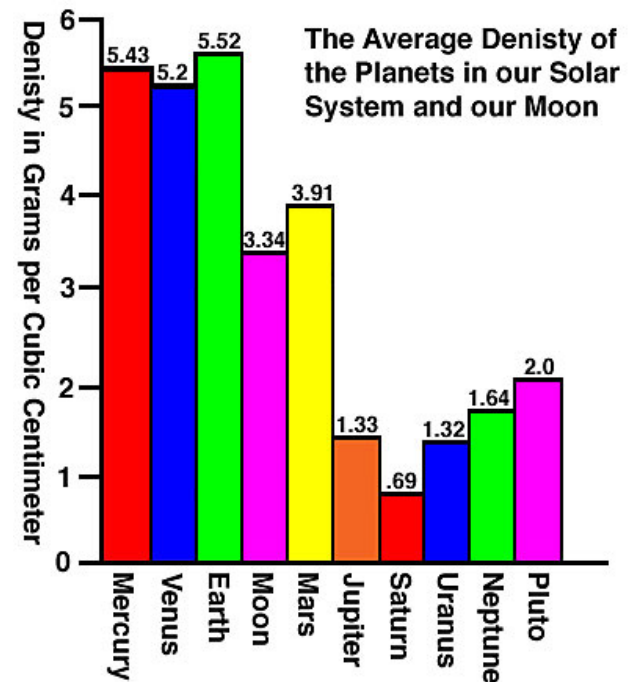- hydrogen and helium are present as gases.

Step 4: Condensation

o   Size and composition of planetesimals depends on temperature and distance from Sun.

o   *Inner solar system*
-   o   Within frost line, only rock and metals can condense.
-   o   Planetesimals therefore made of rock and metals.
-   o   Constitute ~ 0.6% of available material by mass.
-   o   Inner planetismals therefore grew more *slowly*.
-   o   Inner planets are therefore *smaller*.

o   *Outer solar system*
-   o   Beyond frost line, rock, metals and ices condensed.
-   o   Planetesmals therefore contain these materials.
-   o   Constitute ~ 2% of available material by mass.
-   o   Outer planetismals therefore grew more *quickly*.
-   o   Outer planetesmals are therefore *larger*.

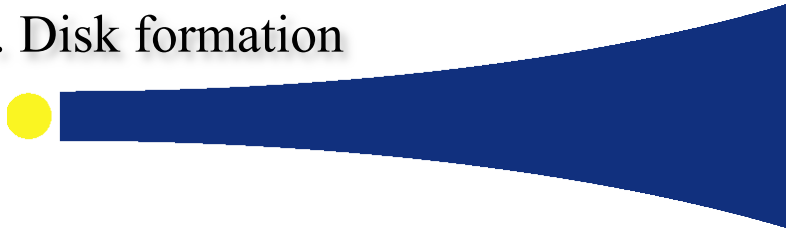o   These process resulted in elementary planetary *cores*.

Step 4: Condensation

o   Densities and distances of objects in solar system supports this condensation theory:

- o Terrestrial planets: 3-6 *g cm⁻³*
  => mainly rocks and metals.

- o Jovian planets: 1-2 *g cm⁻³*
  => more ice and captured gas.

- o Inner Asteroids: contain metalic grains in rocky materials.

- o Outer Asteroids: less metals, and significantly more ice.

The Average Denisty of the Planets in our Solar System and our Moon

Bar chart — y-axis: Denisty in Grams per Cubic Centimeter (0 to 6); x-axis: Mercury, Venus, Earth, Moon, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto.
Values: Mercury 5.43, Venus 5.2, Earth 5.52, Moon 3.34, Mars 3.91, Jupiter 1.33, Saturn .69, Uranus 1.32, Neptune 1.64, Pluto 2.0
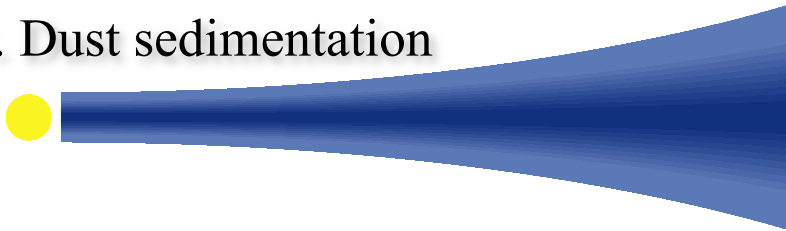
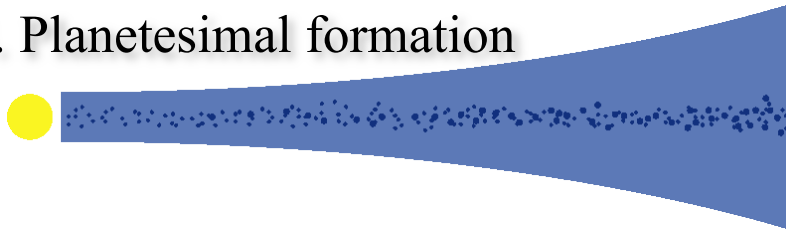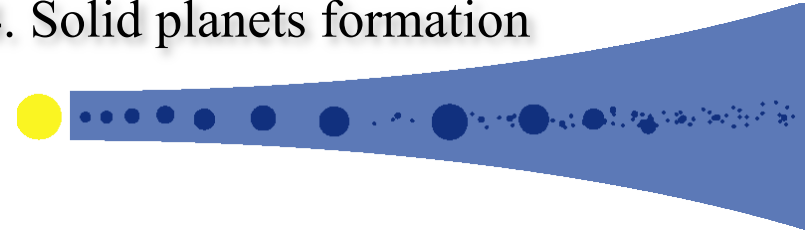# Summary of planet formation in disk

1. Disk formation
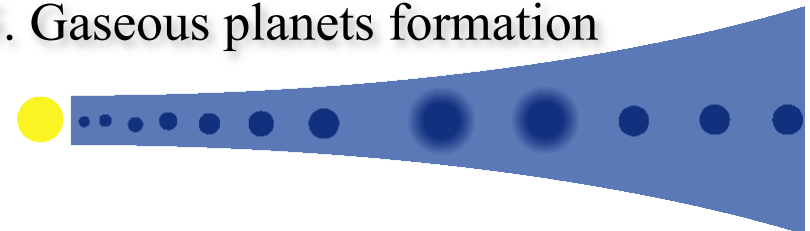
2. Dust sedimentation

3. Planetesimal formation

4. Solid planets formation

5. Gaseous planets formation

6. Disk dissipation
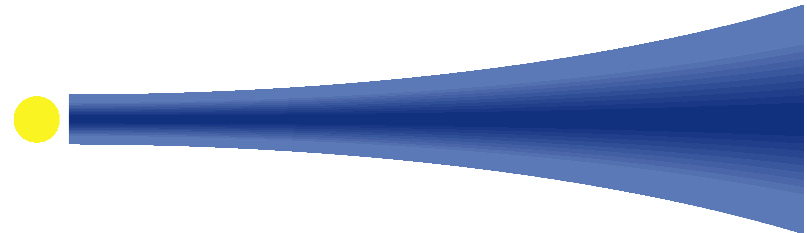
Motion of dust in early disk

o Dust particles experience drag force: $F_{fric} = -\dfrac{4}{3}\rho\sigma c_s v$

    also referred to as Epstien drag force.

o In equilibrium, there is balance between drag and gravity: $F_{fric} = mg$

o Using $g = -\omega^2 z$, $-\dfrac{4}{3}\rho\sigma c_s v = -m\omega^2 z$

$$\therefore v_{term} = -\dfrac{3m\omega^2}{4\rho\sigma c_c} z$$

o Using $\rho = \rho_0 exp(-z^2/H^2)$, the settling time is therefore:

$$\tau_{sett} = \dfrac{z}{v_{term}}$$

$$= \dfrac{\sigma\rho_0}{m\omega^2}\exp\left(-\dfrac{z^2}{2H^2}\right)$$

=> Dust particles settle into the central plane in ~ 1.5×10^5 years.

Planetesimal Formation

o Formation of terrestrial planets from micron-sized dust requires growth by >12 orders of magnitude.

o Useful to divide process into three main phases:

  o **Planetesimal formation:** Planetesimals have radii of order ~10 km. Formed from dust and gas subject to aerodynamic forces, which form rocks-sized objects. Geometric collisions important.

  o **Terrestrial planet formation:** Planetesimals >tens of km interact via gravity. This phase yields terrestrial planets and cores of giant planets. Note, it takes 500 million 10km radius planetesimals to build terrestrial planets.

  o **Giant planet formation:** Once planets have grown to ~$1M_{Earth}$, gravitational coupling with gas becomes significant. At masses >$10M_{Earth}$, interactions become strong enough to capture envelope from propoplanetary

Solar Nebula Hypothesis

(a) Solar nebula contracts to form a spinning disk

(b) Interstellar dust grains act as condensation nuclei allowing accretion of planetesimals

(c) Solar winds push gas out, outer planets already formed

(d) Inner planets start to form from collisions of planetesimals

(e) Collisions continue over time making the 4 inner planets
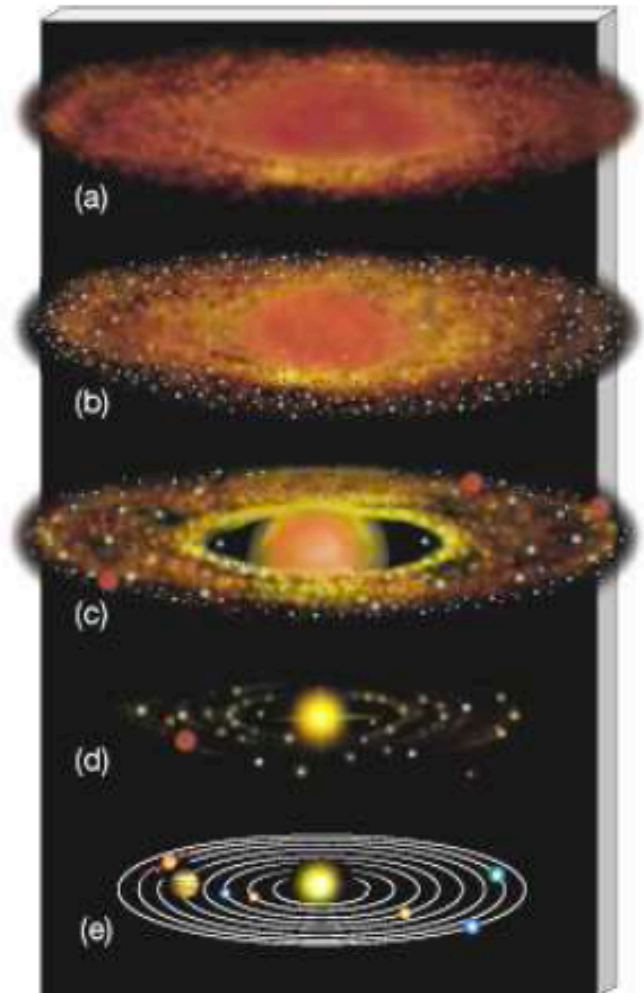
Step 5: Accretion

o   After condensation, growth of solid particles occurs due to collisions.

o   *Accretion* is growth of grains through collisions - the real planet building process.

o   Larger particles formed from both tiny chondrules about 1 mm in size, and from porous molecular aggregates held together by Van der Waals forces.

o   Accretion proceeds in two ways:

1. Collisions due to the *geometric cross section* - direct impacts on 'seed' grain.

2. Collisions due to *gravitational attraction* - sweeping-up of material from a region much larger than grain diameter.
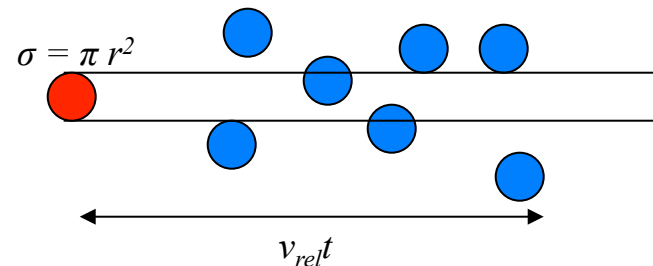
Step 5: Accretion - geometric

o Consider spherical grain of radius r and geometric cross section $\sigma = \pi r^2$. If number of grains m^{-3} is n_g, and relative velocity of the grains is v_{rel} then volume (V) swept out in time t is $V = \sigma v_{rel} t$, (or $V = \sigma v_{rel}$ for 1 second).

o The number of particles (N) encountered in t is $N = V n_g$
$$= \sigma v_{rel} t n_g = \pi r^2 v_{rel} t n_g$$

o In a given period, seed particle's mass grows as

$$\Delta m/\Delta t = m_0 + N m_0 = m_0 (1 + \pi r^2 v_{rel} n_g)$$

where m_0 is the grain mass.

o Mass of the seed particle therefore increases as r^2 *for geometrical collisions.*

$\sigma = \pi r^2$

$v_{rel} t$

Step 5: Accretion - geometric

o Settling and growth of single particle in laminar protoplanetary disk.

o Assumes single particle with initial size = 1 μm (solid), 0.1 μm (dashed), or 0.01 μm (long dashed) accretes all smaller particles it encounters as it settles toward the disk midplane.

o Upper panel shows height above midplane as a function of time, the lower panel the particle radius r.

o See *Astrophysics of Planet Formation* by Armitage.
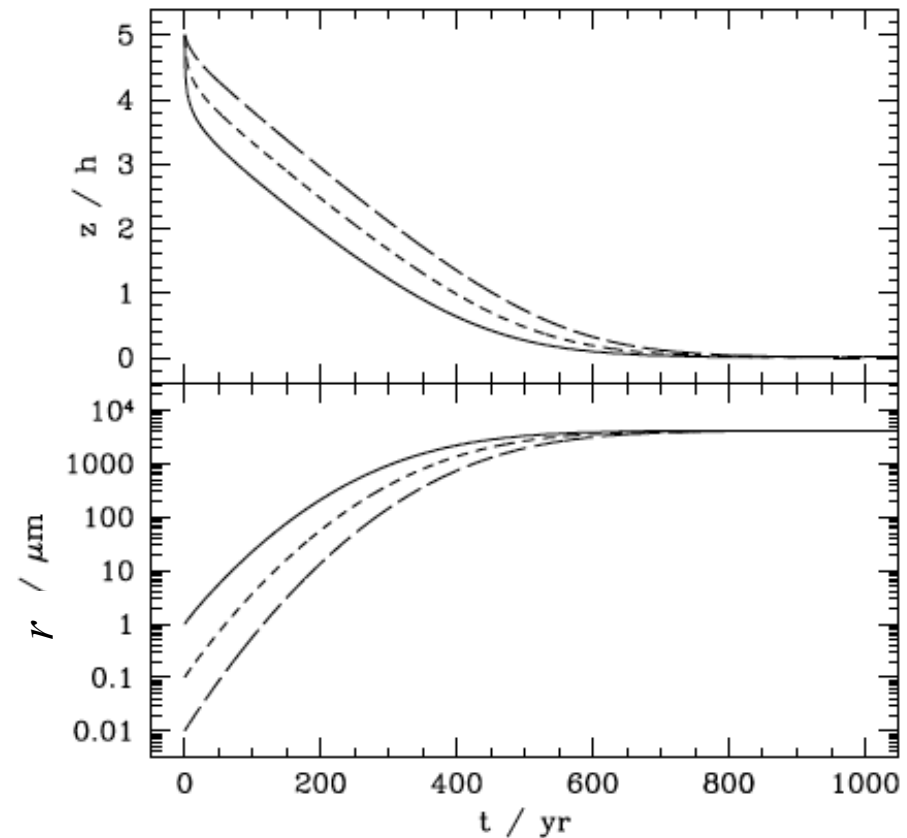
Step 5: Geometric and gravitational accretion

o Objects formed by *geometric accretion are called planetesimals*: act as seeds for planet formation.

o At first, planetesimals were closely packed.

o Then coalesced into larger objects, forming clumps few *km* across in few million years.

o Once planetesimals had grown to few *km*, collisions became destructive, making further growth more difficult.

o *Gravitational accretion* then begins to dominate. This then accretes *planetesimals* to form *protoplanets*.
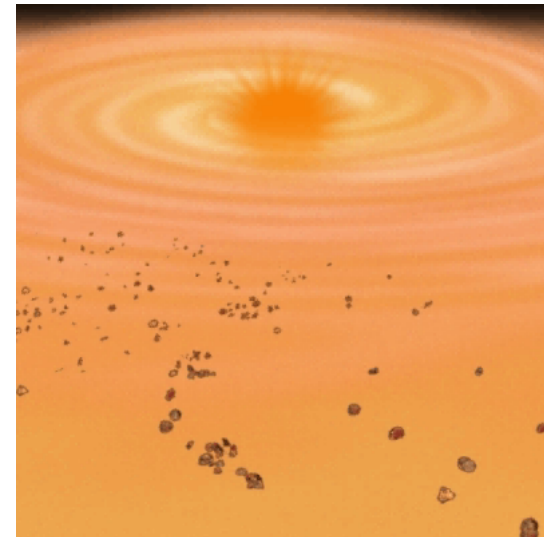
Step 5: Accretion - gravitational

o   When gravity important, grains accrete from larger volume than during geometric growth phase.

o   Consider "test" grain with velocity v_i at a vertical distance s from a "seed" grain. Suppose "test" grain encounters "seed" grain with a final velocity v_f. What is value of s such that the seed grain can capture the "test" grain?

o   Using conservation of angular momentum:

$$mv_f r = mv_i s \qquad \text{Eqn. 1}$$

and conservation of energy:
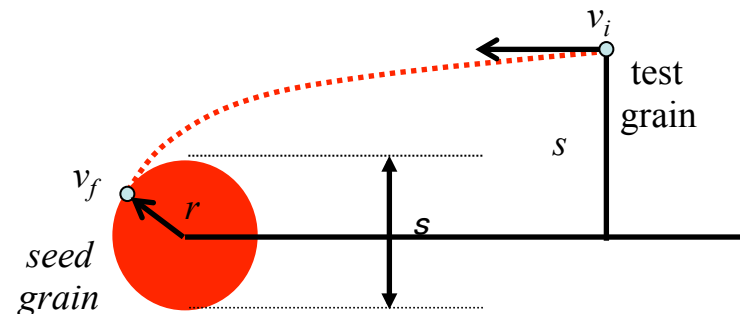
$$1/2mv_i^2 = 1/2mv_f^2 - \frac{GMm}{r} \qquad \text{Eqn. 2}$$

where m is mass of "test" grain and M is mass of "seed" grain.

o   Eliminating v_f from *Eqns. 1* and *2* gives:

$$s^2 = r^2 + \frac{2GMr}{v_i^2} \qquad \text{Eqn. 3}$$

o   As $M \sim r^3 \Rightarrow s^2 \sim r^4$.

Step 5: Accretion - gravitational

o Eqn. 3 can also be written as $s^2 = r^2 + \dfrac{2GMr}{v_i^2}$

$$= r^2\left(1 + \frac{2GM}{rv_i^2}\right)$$

o The escape velocity is $v_{esc} = (2GM/r)^{1/2}$, can write

$$s = r\left(1 + \frac{v_{esc}^2}{v_i^2}\right)^{1/2}$$

o This is the gravitational cross-section, which is much larger than geometric cross-section.

o Results from "gravitaional focussing".

Step 5: Accretion - gravitational

o   The growth rate of the seed particle per unit time is therefore:

$$\Delta m / \Delta t = m_0 (1 + \pi s^2 \, v_{rel} \, n_g)$$

o   As $s^2 \sim r^4 \Rightarrow \Delta m / \Delta t \sim r^4 \Rightarrow runaway \; accretion.$

o   Once grains are large enough that gravity is important, accretion rate increases dramatically.

o   If critical size is achieved, a planetesimal will grow rapidly. Less massive objects grow at a much smaller rate.

o   Model calculation suggest that the first large size objects to form are planetesimals with sizes ~ few tens of km.

5. Accretion

o These processes result in planetesimals of tens of kilometers in size in less than a million years or so.

o Timescale for grows can be calculated from:

$$\tau_{grow} = \left(\frac{1}{m} \frac{dm}{dt} \right)^{-1}$$

$\Delta m / \Delta t \sim r^4$

$\Delta m / \Delta t \sim r^2$

o Not only do bigger planetesimal grow the fastest, but smaller planetesimals are quickly destroyed by fast collisions and turned into smaller fragments => typically one object will dominate a region.
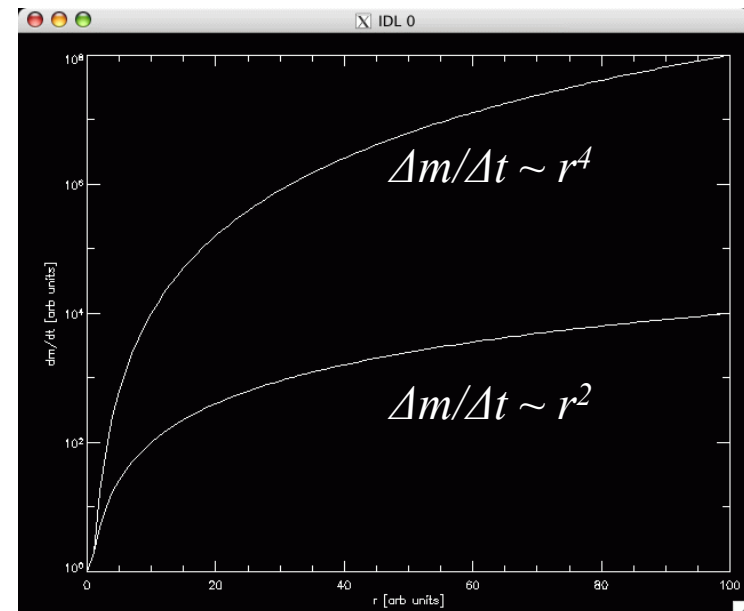
Accretion versus disruption

o   When bodies collide, outcome can be divided into three broad categories:

    o   **Accretion:** All or most of mass of impactor becomes part of mass of final body. Overall, there is net growth.

    o   **Shattering:** Impact breaks up target body, but pieces remain part of single body – a rubble pile.

    o   **Dispersal:** Impact causes fragmentation into many pieces.

o   Boundary between regimes defined vie the specific energy: $Q = \dfrac{1/2 mv^2}{M}$

where *m* is the impactor mass, which collides with a larger body of mass *M,* with a velocity *v*.

o   If Q is high, get dispersal, but if Q is low, get accretion.

Star and Disk Formation Steps

o Class 0: Gravity causes cloud to fragment into dense cores. One core further collapses, becoming hot enough to start emitting in far IR.

o Class I: By Cloud will start to rotate faster and dust will settle in disk. Inner part of cloud will start to fuse hydrogen => star is born. Spectrum does not look like a star yet as is embedded in envelope of gas and dust.

o Class II: Envelope dissipated due to stellar wind. Due to the dense disk, object will still radiate in the IR.

o Class III: Consist of young star surrounded by a transparent disk. Disk matter is discarded through various mechanisms such as photo-evaporation and accretion onto central star.

o Planet formation thought to occur during transition from class II to class III.
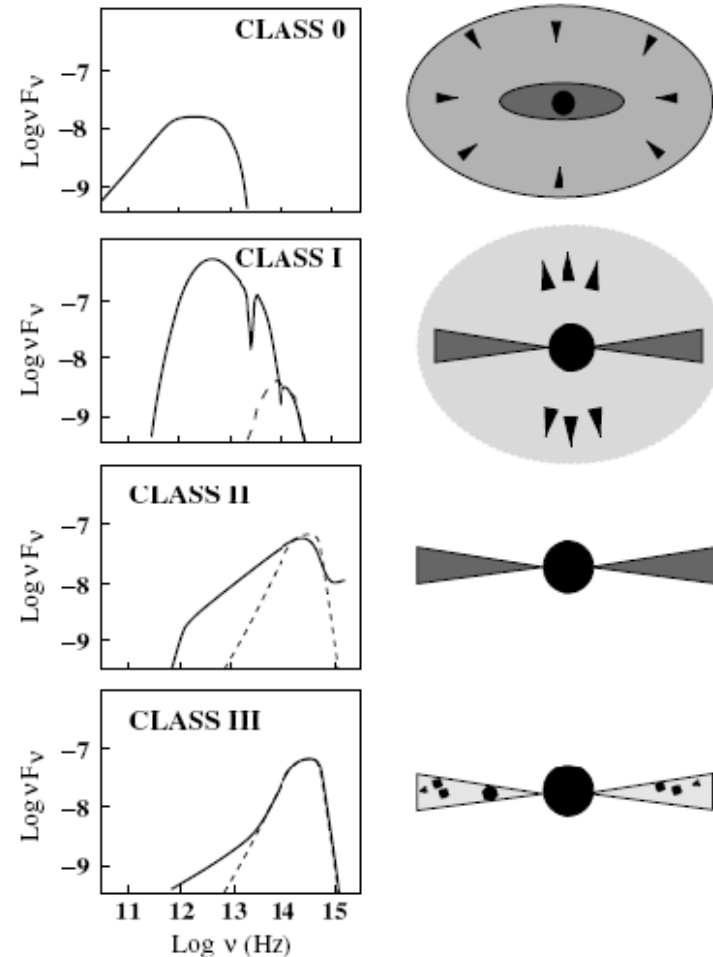
Star and Disk Formation Steps

Class 0
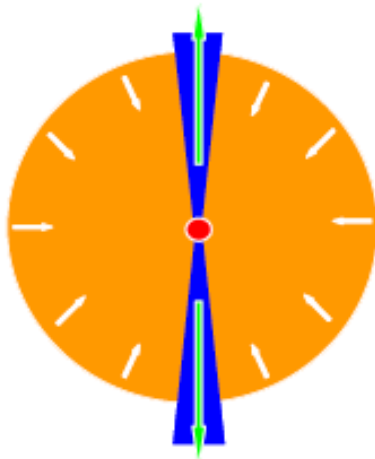
$T \sim 10^4$ years

$R \sim 10^4$ AU

$dM/dt \sim 10^{-5}$ $M_\odot$/yr

Class I
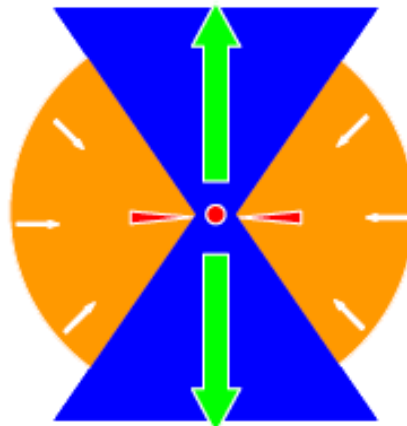
$T \sim 10^5$ years

$R \sim 10^3$ AU

$dM/dt \sim 10^{-6}$ $M_\odot$/yr

Class II

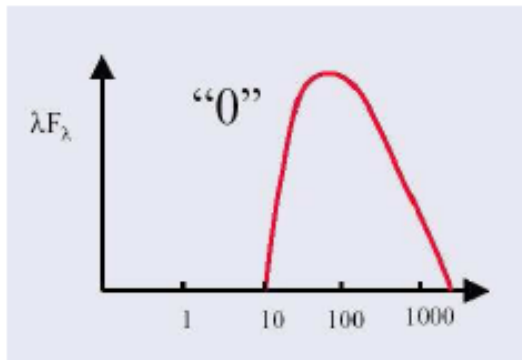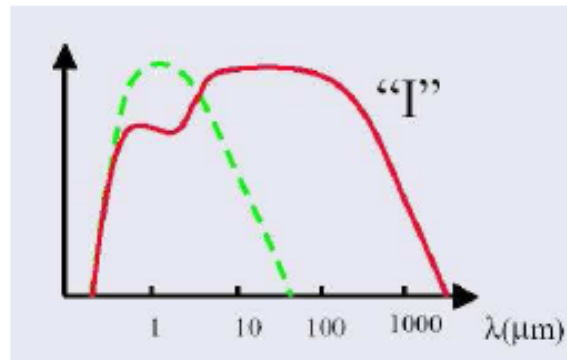$T \sim 10^6$ years

$R \sim 100$ AU

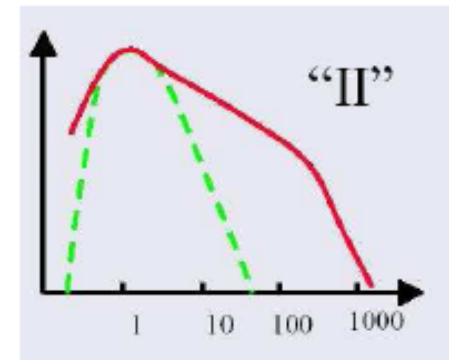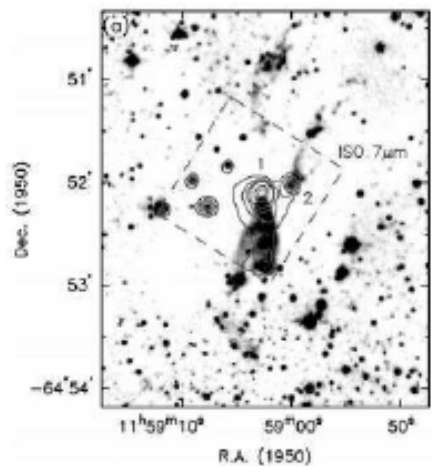$dM/dt \sim 10^{-8}$ $M_\odot$/yr

Star and Disk Formation Steps

BHR 71
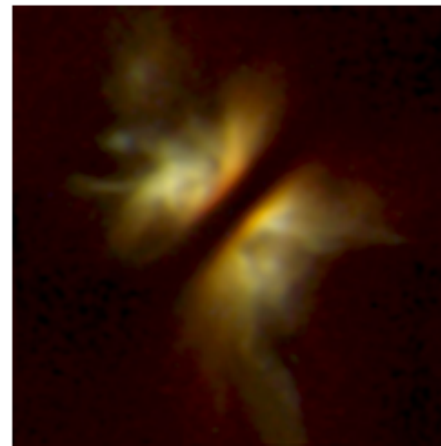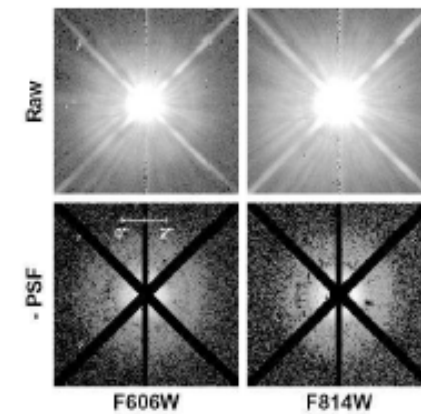
IRAS 04302+2247

TW Hydrae

Bourke 2001

Padgett et al. 1999

Krist et al. 2000

5. Accretion - planet formation

o Accretion therefore progresses according to:

| Geometric Accretion | → | Gravitational Accretion | → | Protoplanets |

o Once planetesimals are formed, the following can occur:

| Planetesimals | → | Protoplanets | → | Planets |

o The final stages in the growth of a Terrestrial planet are dramatic and violent.

o Large Mars-sized protoplanets collide to produce objects such as the Earth and Venus ($M_{Earth} \sim 9\ M_{mars}$).

5. Accretion - the planets

o **Inner Planets**
  - o Formed slowly due to small amount of metals and rocks in early solar nebula.
  - o Geometric accretion rate and gravitational accretion rate small.
  - o By time inner planetesimals were formed and had significant gravitational fields, the nebula had been cleared out by the solar wind.
  - o Then no nebular gas then present to capture an elementary atmosphere.

o **Outer Planets**
  - o Formed less violently.
  - o Great quantities of ice at >3 AU resulted in large rock/ice cores forming.
  - o Reason for rapid core growth is that ices have large cross-sectional area.

5. Accretion - The planetesimal graveyard

o Asteroid belt is 'resting ground' for collision-evolved planetesimals that were not incorporated into a planet.

o Total mass of asteroid belt ~5 x 10^{21} kg (which is about 1/3rd the mass of Pluto or 1/15th the mass of the Moon).

o Ceres the largest asteroid has a diameter of 940 km and a mass of ~10^{21} kg.

o A planet probably did not form in this region because of the rapid formation, and resulting large mass of Jupiter.

The Main Asteroid Belt

Mars

Sun

Earth

Jupiter

(Orbits drawn approximately to scale)

99-1030B-3
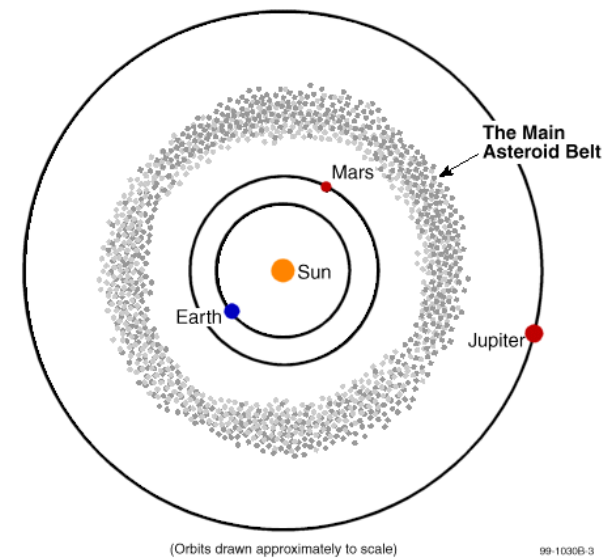
Formation of giants: numerical simulation

o   Three stages for gas giant formation:

    1.   Core accretion.

    2.   Accretion of gas until $M_{gas} \sim M_{core}$

    3.   Runaway accretion of gas
        $=> M_{gas} >> M_{core}$
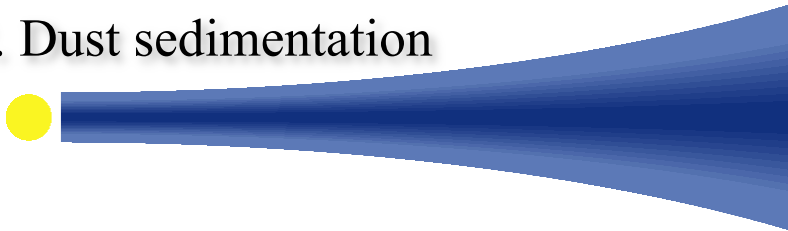
o   See Pollack et al. (1996).
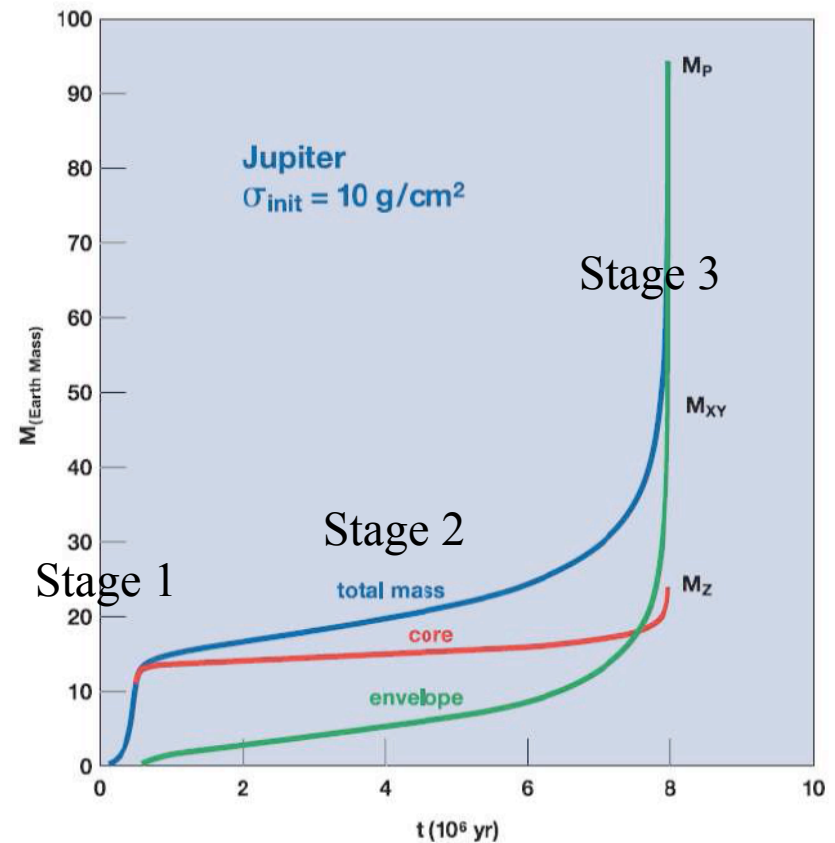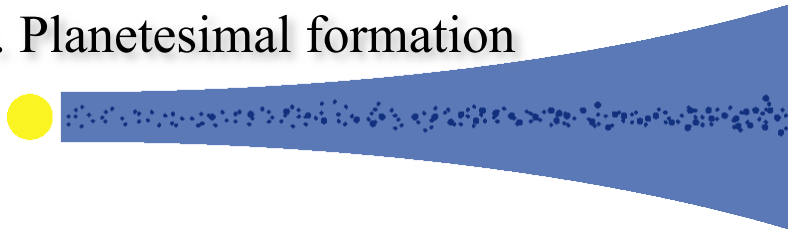
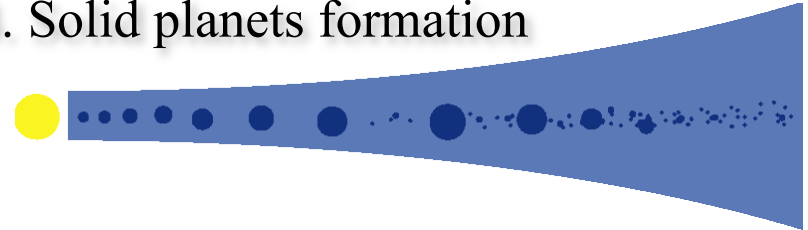# Summary of planet formation in solar nebula theory
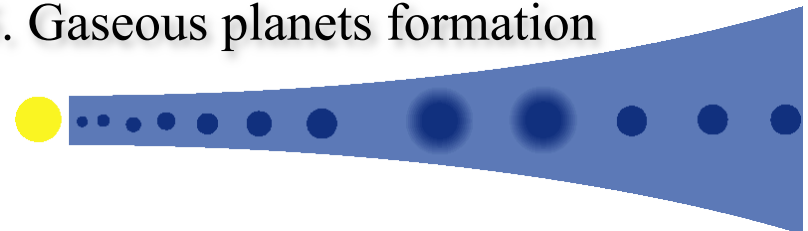
1. Disk formation

2. Dust sedimentation

3. Planetesimal formation

4. Solid planets formation

5. Gaseous planets formation

6. Disk dissipation