Introduction to Plasma Physics (PY5012)
Lectures 1 & 2: Basic Concepts

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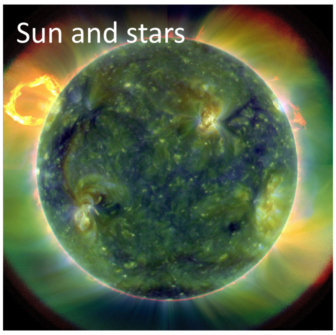
What is a plasma?

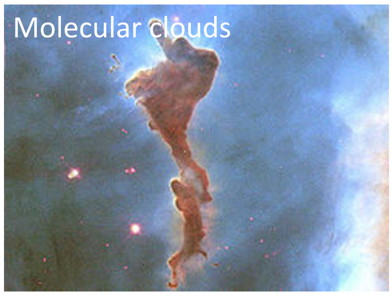
- "A plasma is a quasi-neutral gas consisting of positively and negatively charged particles (usually ions and electrons) which are subject to electric, magnetic and other forces, and which exhibit collective behaviour."
  - Schwartz, Owen and Burgess, "Astrophysical Plasmas".

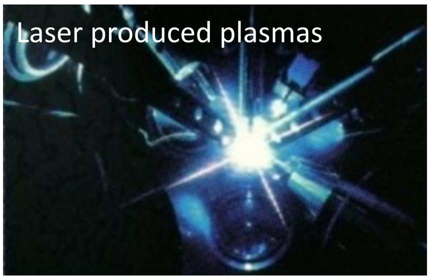
- Langmuir in 1920s showed that an ionised gas can support oscillations, which resemble a jelly-like substance. He named it a "plasma".

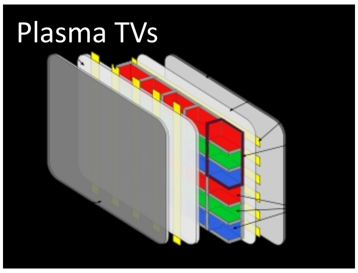
- Scientific term for "plasma" was introduced in 1839 by Czech biologist to describe a jelly-like medium of cells.

Plasmas in Nature and Technology

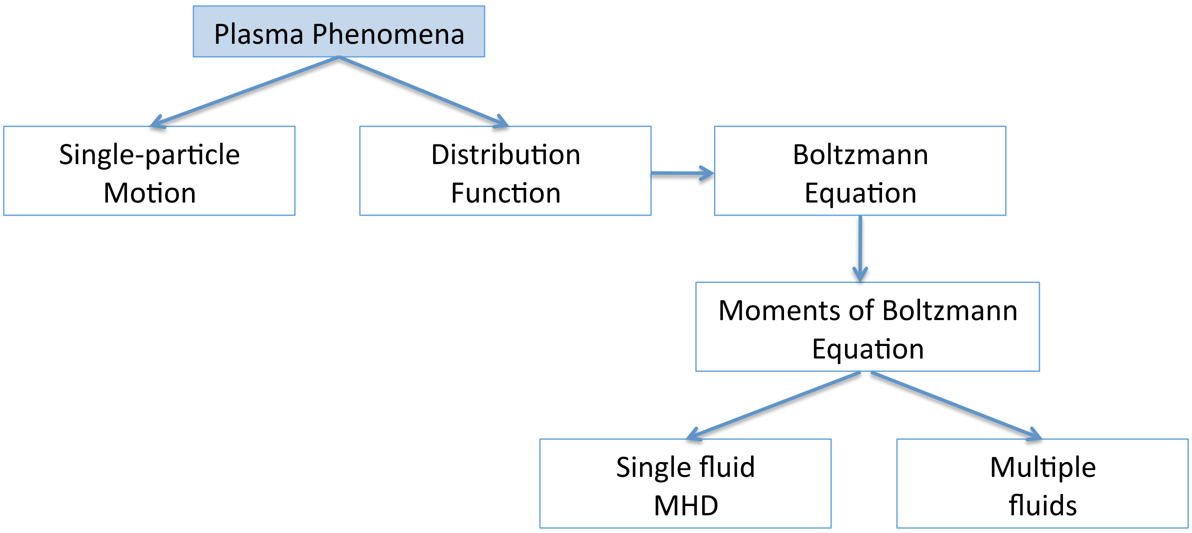








Hierarchy of plasma phenomena



Basic Parameters: Speed, Energy and Temperature

- For ensemble of N particles of mass m and velocity u, average energy per particle is

$$\langle E \rangle = \frac{1}{2N}\sum_{i=1}^{N} m_i u_i^2$$

- In thermal equilibrium, particles have Maxwell-Boltzmann distribution of speeds.

$$f(u) = N\sqrt{\frac{m}{2\pi k_B T}}e^{-1/2mu^2/k_BT}$$

- Average KE can be calculated using

$$\langle E \rangle = \frac{\int_{-\infty}^{+\infty} 1/2mu^2 f(u)du}{\int_{-\infty}^{+\infty} f(u)du}$$

- Integrating numerator by parts, and denominator using $\int_{-\infty}^{+\infty} e^{-a^2x^2}dx = \sqrt{\pi}/a$ we have $\langle E \rangle = 1/2k_BT$ or in 3D,

$$\langle E \rangle = 3/2k_BT$$

Basic Parameters: Speed, Energy and Temperature

- Using $<E> = 3/2\ k_BT$, average KE of a gas at1 K is ~2.07 x 10^{-23} J.

- Plasma temperature often given in electron Volts (eV), where 1 eV = 1.602 x 10^{-19} Coulombs (C).

- For electron or proton, $|q|$ = 1.602 x 10^{-19} C, therefore

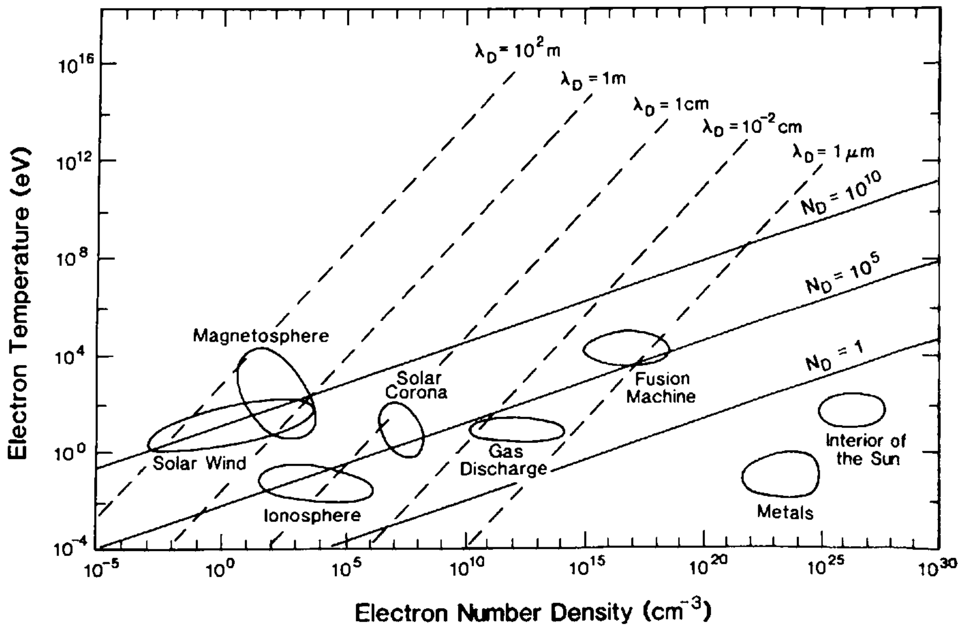
$$E = \frac{1/2mu^2}{2|q|} \quad \text{eV}$$

- **Note:** Typically, energy corresponding to k_BT used for temperature, where k_BT = 1 eV = 1.602 x 10^{-19} J.

- Q. Show that a 0.5 eV plasma corresponds to a temperature of 5,800 K (solar photosphere).

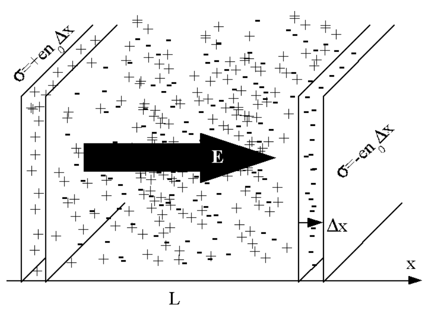
Range of plasma parameters

- Density range: >30 orders of magnitude. Temperature range: ~10 orders.



Plasma Oscillations and Plasma Frequency

- Consider plasma of equal number of positive and negative charges. Overall, plasma is neutral, so $n_e = n_i = n$
- Now displace group of electrons by Δx.
- Charge separation gives $\boldsymbol{E}$, which accelerates electrons towards initial position. Electrons overshoot equilibrium position.



- Using Newton's Law, $m_e \dfrac{d^2\Delta x}{dt^2} = eE$ *(1)*
- Displacement sets up E across distance L, similar to parallel plate capacitor.
- Charge per unit area of slab is $\sigma = -ne\Delta x \Rightarrow E = \sigma / \varepsilon_0 = -ne\Delta x / \varepsilon_0$

Plasma Oscillations and Plasma Frequency

- Therefore, Eqn. (1) can be written $$\frac{d^2\Delta x}{dt^2} = -\frac{ne^2}{m_e\varepsilon_0}\Delta x = -\omega_p^2\Delta x$$

  where $$\omega_p^2 = \frac{ne^2}{m_e\varepsilon_0}$$ *Electron Plasma Frequency*

- Plasma oscillations are result of plasma trying to maintain charge neutrality.

- Plasma frequency commonly written $$f_p = \frac{\omega_p}{2\pi} = 9000\sqrt{n_e} \text{ Hz}$$ where n_e is in cm^{-3}.

- In Solar System, f_p ranges from hundreds of MHz (in solar corona) to <1 kHz (near outer planets).

- What is plasma frequency of Earth's ionospere? What implication does this have for (i) short wave radio communications and (ii) radio astronomy?

Plasma Criteria

- In a partially ionized gas where collisions are important, plasma oscillations can only develop if mean free time between collisions (τ_c) is long compared with the oscillation period ($\tau_p=1/\omega_p$).

- That is, $\tau_c >> \tau_p$ or $\tau_c/\tau_p >> 1$ *Plasma Criterion #1*

- Above is *a criterion* for an ionized gas to be considered a plasma.

- Behaves like neutral gas is criterion is not true.

- Plasma oscillations can be driven by natural thermal motions of electrons ($E=1/2k_BT_e$). Work by displacement of electron by Δx is and using $E = -ne\Delta x / \varepsilon_0$

$$\begin{aligned} W &= \int F dx \\ &= \int_0^{\Delta x} eE(x)dx \\ &= \frac{e^2 n\Delta x^2}{2\varepsilon_0} \end{aligned}$$

Plasma Criteria

- Equating work done by displacement with average energy in thermal agitation

$$\frac{e^2 n \Delta x^2}{2\varepsilon_0} \approx \frac{1}{2} k_B T_e$$

- The maximum distance an electron can travel is $\Delta x_{max} = \lambda_D$, where

$$\boxed{\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n}}} \qquad \textit{Debye Length}$$

- Gas is considered a plasma if length scale of system is larger than Debye Length:

$$\boxed{\lambda_D << L} \qquad \textit{Plasma Criterion \#2}$$

- Debye Length is spatial scale over which charge neutrality is violated by spontaneous fluctuations.

- Debye Number is defined as $N_D = n\,[4\pi\lambda_D^3/3]$

Debye Shielding

- Plasmas do not contain strong electric fields as they reorganize to shield from them.

- Plasma oscillations are excited to assert *macroscopic* neutrality.

- If plasma subjected to external ***E***, free charges redistribute so that plasma is shielded.

- Suppose immerse test particle $+Q$ within a plasma with $n_i = n_e = n$

- At $t = 0$, electric potential is $\Phi(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

- As time progresses, electrons are attracted, while ions are repelled. As $m_i >> m_e$ neglect motion of ions.

- At $t >> 0$, $n_e > n$ and a new potential is set up, with charge density:

$$\rho = e(n_e - n_i).$$

Debye Shielding

- New potential evaluated using Poisson's equation:

$$\nabla^2\Phi(r) = -\frac{\rho}{\varepsilon_0} = -\frac{e(n_e - n_i)}{\varepsilon_0}$$

- In presence of potential, electron number density is

$$n_e(r) = ne^{-e\Phi(r)/k_B T_e}$$

- Subbing this into Poisson's equation in spherical coordinates.

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = -\frac{en}{\varepsilon_0}\left[e^{-e\Phi/k_B T_e} - 1\right]$$

- For $|e\Phi| << k_B T_e$ can us Taylor expansion: $e^x \approx 1 + x$ =>

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) \approx \left[\frac{ne^2}{\varepsilon_0 k_B T_e}\right]\Phi(r) = \frac{1}{\lambda_D^2}\Phi(r)$$

  where λ_D is the *Debye shielding length.*

Debye Shielding

- Solution to previous is $\Phi(r) = \left[\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\right]e^{-r/\lambda_D}$

- As r -> 0, potential is that of a free charge in free space, but for $r >> \lambda_D$ potential falls exponentially.

- Coloumb force is long range in free space, but only extends to Debye length in plasma.

- For positive test charge, shielding cloud contains excess of electrons.

- Recall $\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n}}$

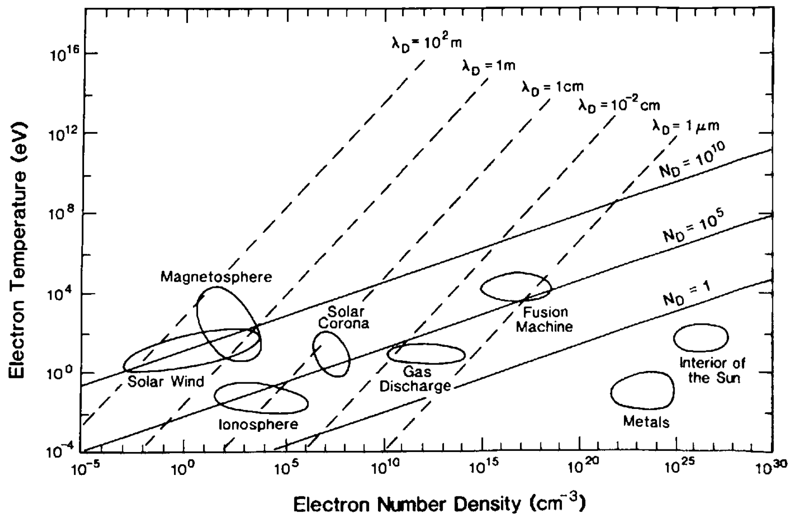
  => size of shielding cloud increases as electron temperature as electrons can overcome Coulomb attraction. Also, λ_D is smaller for denser plasma because more electrons available to populate shielding cloud.

Debye Shielding

- Useful numerical expression for Debye length:

$$\lambda_D \cong 69\sqrt{T_e / n}$$

where T_e is in K and n is in m^{-3}.



The Plasma Parameter

- The typical number of particles in a Deybe sphere is given by *the plasma parameter:*

$$\Lambda = 4\pi n \lambda_D^3$$
$$= \frac{1.38 \times 10^6 T_e^{3/2}}{n^{1/2}}$$

- Defined as N_D in Inan & Gollowski and usually given as argument of *Coulomb Logarithm* (log(Λ)).

- If Λ<<1, the Debye sphere is sparsely populated, corresponding to a strongly coupled plasma.

- Strongly coupled plasmas tend to be cold and dense, whereas weakly coupled tend to be diffuse and hot.

The Plasma Parameter

Description	Plasma parameter magnitude	
	Λ<<1	Λ>>1
Coupling	Strongly coupled plasma	Weakly coupled plasma
Debye sphere	Sparsely populated	Densely populated
Electrostatic influence	Almost continuously	Occasional
Typical characteristic	Cold and dense	Hot and diffuse
Examples	Solid-density laser ablation plasmas Very "cold" "high pressure" arc discharge Inertial fusion experiments White dwarfs / neutron stars atmospheres	Ionospheric physics Magnetic fusion devices Space plasma physics Plasma ball

Collisions

- Neutral particles have quite small collision cross sections. As Coulomb force is long range, charged particles are more frequent.

- A *collisional plasma* is one where $\lambda_{mfp} << L$

  where L is the observational length scale and $\lambda_{mfp} = 1/\sigma n$ is the mean free path.

- The effective Coulomb cross-section is $\sigma = \pi r_c^2$

- An electron will be affected by a neighbouring ion if the Coulomb potential is of the order of the electron thermal energy:

$$\frac{e^2}{4\pi\varepsilon_0 r_c} \approx \frac{3}{2}k_B T$$

$$\Rightarrow \sigma = \pi\left(\frac{e^2}{6\pi k_B \varepsilon_0}\right)^2 \frac{1}{T^2}$$

- At $T = 10^6$ K, $\sigma \sim 10^{-22}$ m^2, which is much larger than the geometric nuclear cross-section of 10^{-33} m^2.

Collisions

- In terms of the plasma parameter, the collision frequency ($\nu = n\, e\, v$) is

$$\nu \approx \frac{\omega_p}{64\pi} \frac{\ln \Lambda}{\Lambda}$$

where ln(Λ) is the Coulomb logarithm. Used as Λ is large, but $10 < \ln(\Lambda) < 30$.

- In a weakly coupled plasma, $\nu << \omega_p$ => collisions to not effect plasma oscillations

- More rigorously, it can be shown that

$$\nu \approx \frac{\sqrt{2}\omega_p^4}{64\pi n_e} \left(\frac{k_B T}{m_e} \right)^{-3/2} \ln \Lambda$$

- Thus, diffuse, high temperature plasmas tend to be collisionless. See page 156 of Inan & Golkowski.

Examples of key plasma parameters

	$n(m^{-3})$	T(eV)	$\omega_p(sec^{-1})$	$\lambda_D(m)$	Λ
Interstellar	10^6	10^{-2}	6×10^4	0.7	4×10^6
Solar Chromosphere	10^{18}	2	6×10^{10}	5×10^{-6}	2×10^3
Solar Wind (1AU)	10^7	10	2×10^5	7	5×10^{10}
Ionosphere	10^{12}	0.1	6×10^7	2×10^{-3}	1×10^5
Arc discharge	10^{20}	1	6×10^{11}	7×10^{-7}	5×10^2
Tokamak	10^{20}	10^4	6×10^{11}	7×10^{-5}	4×10^8
Inertial Confinement	10^{28}	10^4	6×10^{15}	7×10^{-9}	5×10^4

Table 1.1: *Key parameters for some typical weakly coupled plasmas.*

* From *Plasma Physics* by R. Fitzpatrick

Next Lecture: Single-particle Motion