Introduction to Plasma Physics (PY5012)
Homework 3

Submit solutions by email on Wednesday, December 21, 2011
Note: This homework is worth 10% of total module mark

1. Starting from the linearized solenoidal constraint ($\nabla\cdot\mathbf{B}_1 = 0$), show that if this is true at one time then it is also true at any subsequent time.
   [Hint: consider the linearized induction equation]

2. Calculate the Alfvén wave fundamental oscillation period in a coronal loop of length 50 Mm with background magnetic field of 10^{-3} T and particle density of 10^{15} m^{-3}.
   [Hint: fundamental oscillation modes have $\lambda = L/2$]

3. Starting from the generalized wave equation,

$$\frac{\omega^2 \mathbf{v}_1}{v_\mathrm{A}^2} = k^2 \cos^2\left(\theta_{\mathbf{kB}_0}\right)\mathbf{v}_1 - \left(\mathbf{k}\cdot\mathbf{v}_1\right)k\cos\left(\theta_{\mathbf{kB}_0}\right)\hat{\mathbf{B}}_0 + \left[\left(1+\frac{c_\mathrm{s}^2}{v_\mathrm{A}^2}\right)\left(\mathbf{k}\cdot\mathbf{v}_1\right) - k\cos\left(\theta_{\mathbf{kB}_0}\right)\left(\hat{\mathbf{B}}_0\cdot\mathbf{v}_1\right)\right]\mathbf{k}$$

   derive the magnetoacoustic dispersion relation using two instances of scalar multiplication,

$$\omega^4 - \omega^2 k^2\left(c_\mathrm{s}^2 + v_\mathrm{A}^2\right) + c_\mathrm{s}^2 v_\mathrm{A}^2 k^4 \cos^2\theta_{\mathbf{kB}_0} = 0$$

   [Hint: consider which two vectors have the simplest self dot-product]

4. Calculate the magnetoacoustic fast-mode wave phase speed in a 4 MK stellar corona with a background particle density of 10^{14} m^{-3}, Consider a horizontal wave propagating at right angles to a background magnetic field of 10^{-3} T.
   [Hint: use a mean molecular weight, μ, of 0.59]

5. Discuss how the solution to Q4 and the wave propagation scenario would change when considering that magnetic field lines are not purely vertical in coronae.