Projet CNEPRU 2013
Matrix Field Theory and Supersymmetric Gauge Theory

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1 Programme de Recherche

Axis 1: Matrix Field Theory

- The renormalization group method in matrix models and noncommutative field theory.
- Emergent phenomena (geometry, gauge theory and gravity) in Yang-Mills matrix theories.

Axis 2: Supersymmetric Gauge Theory

- The Seiberg-Witten-Nekrasov solution of $\mathcal{N} = 2$ supersymmetric gauge theories in $d = 4$ and instanton calculus.

2 État de la Question et Méthodologie

AXIS 1: Matrix Field Theory Historically matrix models played a central role in the study of quantum chromodynamics (QCD) and two-dimensional quantum gravity (non-critical strings and random surfaces). The large N limit is obviously one of the most powerful tools available in the matrix models which are used in attempting to understand these strongly coupled systems.

In recent years it has also become clear that matrix models will play a leading role in understanding non-commutative and fuzzy field theories non-perturbatively. There are two classes of matrix models of interest to us in this proposal. The first class is scalar matrix field theory which should be viewed as a non-perturbative regulator of non-commutative scalar field theory. The questions of central importance regrading this class is renormalizability and phase structure. The second class of matrix models of interest to us here are Yang-Mills matrix models which have been shown to exhibit a remarkable phenomena of emergent geometry, emergent gauge theory and emergent gravity.

The main analytical tool to be used in studying the above matrix models is the $1/N$ expansion. The renormalization group method is a very powerful analytical tool which should also be developed for matrix models and no-commutative field theories. This will be in fact one of the central activity during this project. The two approaches we aim at studying is the functional renormalization group and the Higuchi et al approach.

The multi-trace approach is also another powerful analytical technique which can also be brought to bear on the problem of the phase structure of scalar matrix models. Other field and matrix theories techniques will be explored as well.

At this stage we envisage the use of the Monte Carlo method to study the phenomena of emergent geometry and emergent gauge theory in Yang-Mills matrix models in various dimensions. The impact of supersymmetry on emergent geometry and the stability of four dimensional emergent spaces will be the second central questions we aim at answering in this project.

The phenomena of emergent gravity (and possibly emergent cosmology and emergent time) is much more complicated and requires a thorough theoretical study first using for example ordinary perturbation theory of matrix/field theory.

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AXIS 2: Supersymmetric Gauge Theory The low energy physics of quantum chromodynamics, which is an example of a non-abelian gauge theory, is highly and intrinsically non-perturbative. Some of the most important low energy properties of QCD are confinement, the mass gap and spontaneous chiral symmetry breaking which have no satisfactory theoretical explanation.

In recent years two things made it possible to solve exactly a certain class of non-abelian gauge theories (the $\mathcal{N} = 2$ supersymmetric theories) and thus probe directly their low energy physics. Firstly the ADHM (Atiyah, Drinfeld, Hitchin and Manin) construction of all multi-instantons contributing to the semi-classical approximation of the path integral. Secondly supersymmetry which gives rise to a major cancellation between fermionic and bosonic contributions which simplifies considerably the integrand.

As it turns out the holomorphy of the Lagrangian density of a supersymmetric non-abelian gauge theory is a very powerful tool in extracting non-perturbative exact results with very little detailed calculations. Indeed in 1994 Seiberg and Witten derived explicitly the low energy Wilsonian effective action of $\mathcal{N} = 2$ supersymmetric non-abelian gauge theory in four dimensions using essentially the holomorphy property (Seiberg-Witten curves).

In much more recent years $(2003 - 2004)$ Nekrasov derived the same result of Seiberg and Witten by means of the instanton calculus together with the Localization technique and an appropriate Lorentz deformation of the action. This is the first explicit solution of a non-abelian gauge theory in 4 dimensions starting from a microscopic action and as such it is a huge step towards understanding QCD.

In this project we aim at reviewing as much as possible of this work so that we can develop a new research axis along these lines.

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