Particle acceleration at relativistic shock waves

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Why relativistic Fermi acceleration?

→ Relativistic outflows produce relativistic shock waves...
→ Dissipation in the shock transition is very efficient...
→ Fermi acceleration is expected to produce powerlaws
  ⇒ non-thermal radiation spectra...
→ Shock acceleration physics is contained a small number of parameters...

Quantities of interest:

→ spectral index of powerlaw
→ maximal energy (← acceleration timescale)
→ energy content
I. Preliminary considerations
   a) Formation of a collisionless relativistic shock
   b) Hydrodynamics of a relativistic shock front

II. Relativistic Fermi acceleration in the test particle limit
   a) Particle kinematics and energy gain
   b) Inhibition of Fermi acceleration in superluminal shock waves

III. Modern relativistic Fermi acceleration
   a) Hints from particle-in-cell simulations
   b) Generation of micro-turbulence by accelerated particles
   c) Generalizations

IV. Application to astrophysics: gamma-ray burst external shock waves
   a) Gamma-ray burst afterglows
   b) Gamma-ray burst afterglow in magnetized circumburst media
**Formation of a ultra-relativistic collisionless shock**

**Collisionless shock:** at densities typical of the ISM, the Coulomb collision time $\tau_{\text{Coul}}$ far exceeds the dynamical timescale $R/\gamma_{sh}c$ (shock frame): $\tau_{\text{Coul}} \sim 10^{31} \text{cm} \, \gamma_{sh}^{-1} \ldots$

$\Rightarrow$ fluctuating electromagnetic fields act as the agents of the shock transition, with a typical length scale $\sim c/\omega_p \sim 10^7 \text{ cm} \, n^{-1/2}$

**Shock formation:** in the simplest case, the interpenetration of two unmagnetized flows gives rise to beam instabilities, that isotropize the particles, dissipating the incoming kinetic energy into thermal energies.

![Diagram of shock formation](image)

Dieckmann et al. 08

Standard instability: filamentation mode (aka Weibel instability), e.g. Medvedev & Loeb 99
(Hydro-)Dynamics of relativistic shock waves

Outflow of proper density $n_{ej}$ moving at Lorentz factor $\gamma_{ej}$ relative to medium of density $n_{ext}$

Momentum flux balance between (cold) incoming flows:

\[ \gamma_{b|ej}^2 \beta_{b|ej}^2 n_{ej} = \gamma_{b|ext}^2 \beta_{b|ext}^2 n_{ext} \]

\[
\begin{cases}
\gamma_{ej} \gg \sqrt{n_{ej}/n_{ext}} \Rightarrow \text{ultra-relativistic reverse shock}, & \gamma_{b|ext} \approx \frac{1}{\sqrt{2}} \gamma_{ej}^{1/2} \left( \frac{n_{ej}}{n_{ext}} \right)^{1/4}, \quad \gamma_{sh} \approx \sqrt{2} \gamma_{b|ext} \\
\gamma_{ej} \ll \sqrt{n_{ej}/n_{ext}} \Rightarrow \text{non-relativistic reverse shock}, & \gamma_{b|ext} \approx \gamma_{ej}, \quad \gamma_{sh} \approx \sqrt{2} \gamma_{ej}
\end{cases}
\]

(assuming ultra-relativistic forward shock: $\gamma_{b|ej}^2 \gg n_{ext}/n_{ej}$)
(Hydro-)Dynamics of relativistic shock waves

Outflow of proper density $n_{ej}$ moving at Lorentz factor $\gamma_{ej}$ relative to medium of density $n_{ext}$

**Continuity equations (strong, unmagnetized shock) with velocities $\beta_+, \beta_-$ in shock frame:**

\[
\begin{align*}
\beta_+ \gamma_+ n_+ & = \beta_- \gamma_- n_- \\
\beta_+^2 \gamma_+^2 (e_+ + p_+) + p_+ & = \beta_-^2 \gamma_-^2 (e_- + p_-) + p_- \\
\beta_+^2 (e_+ + p_+) & = \beta_-^2 (e_- + p_-) \\
\end{align*}
\]

$\Rightarrow$

\[
\begin{align*}
\beta_{sh|b} & \approx \frac{1}{3} \\
\gamma_{sh} & \approx \sqrt{2} \gamma_{b|ext} \\
n_b & \approx 4 \gamma_{b|ext} n_{ext} \\
e_b & \approx 4 \gamma_{b|ext}^2 \rho_{ext} c^2
\end{align*}
\]

*E.g. Blandford & McKee 76, Kirk & Duffy 99*
Schematics of gamma-ray burst blast waves

**Blast wave geometry**

\[ \Delta_b \sim c t (\beta_{sh} - \beta_b) \sim c t / (4\gamma_b^2) \]

\[ (\gamma_{sh} \sim \sqrt{2}\gamma_{b|ext}, \quad \gamma_b \equiv \gamma_{b|ext}) \]
...once the shock is formed, its dynamics as the physics of particle acceleration is controlled by a few number of parameters, most notably:

- the shock Lorentz factor $\gamma_{sh}$
- the upstream magnetization $\sigma$

\[ \sigma \equiv \frac{B_u^2}{4\pi \rho_u c^2} = \left( \frac{v_A}{c} \right)^2 = 1/M_A^2 \]

**pulsar winds**
- $\gamma \sim 10^3$ - ...
- $\sigma \sim 0.001$ - ...?

**powerful active galactic nuclei**
- $\gamma \sim 10-50$
- $\sigma \sim \text{low} \rightarrow \text{high}$

**gamma-ray bursts**
- $\gamma \sim 100$
- $\sigma \sim 10^{-9}$ - ?
1st order Fermi acceleration

as the particle circulates upstream and downstream, it systematically experiences a converging flow when returning to the shock $\Rightarrow$ head-on collisions

$\beta_{\text{down}} \approx 1/3$

$\gamma_{\text{b}} \approx \gamma_{\text{sh}}/\sqrt{2}$
Fermi acceleration in three steps

Step 1: particle interacts with scattering centers (=magnetic inhomogeneities); in the rest frame of the scattering center, diffusion is elastic: $E_f = E_i$

Step 2: follow a cycle around the shock: upstream $\rightarrow$ downstream $\rightarrow$ upstream

2.a: shock crossing from up- to downstream

$$E_{\text{down}}^{\text{in}} = \gamma_b E_{\text{up}}^{\text{out}} (1 - \mu_{\text{up}}^{\text{out}} \beta_b)$$

$\beta_b, \gamma_b$: velocity between up and down

cosine of angle to shock normal (= velocity component along shock normal)

2.b: particle bounces back downstream

$$E_{\text{down}}^{\text{out}} = E_{\text{down}}^{\text{in}}, \quad \mu_{\text{down}}^{\text{out}} \neq \mu_{\text{down}}^{\text{in}}$$

(elastic scattering,

$\mu_{\text{down}}^{\text{in}}$ Lorentz transform of $\mu_{\text{up}}^{\text{out}}$)

2.c: particle crosses the shock toward upstream (if it does not escape)

$$E_{\text{up}}^{\text{in}} = \gamma_b E_{\text{down}}^{\text{out}} (1 + \mu_{\text{down}}^{\text{out}} \beta_b)$$

finally, rewrite for cycle n to n+1: $E_{\text{up}}^{\text{out}} \equiv E_{\text{up}}^{n}, \quad E_{\text{up}}^{\text{in}} \equiv E_{\text{up}}^{n+1}$
Fermi acceleration in three steps

Step 3: collect the results and average over angles (isotropic scattering)

$$\frac{E_{up}^{n+1}}{E_{up}^n} = \gamma_b^2 \left( 1 + \mu_{down}^{out} \beta_b \right) \left( 1 - \mu_{up}^{out} \beta_b \right)$$

if isotropic populations:

$$\frac{E_{up}^{n+1}}{E_{up}^n} \sim \gamma_b^2$$

particle gains energy at each cycle, but has a non zero probability of escaping from the flow

accelerated population: sum of populations escaped at each cycle

$$\Rightarrow$$ power law spectrum $f(E) \propto E^{-s}$
Fermi acceleration at ultra-relativistic shock waves

- Crucial difference w.r.t. non-relativistic Fermi: shock wave velocity $\sim c \sim$ particle velocity

  ... for example: $\gamma_{sh} = 100 \Rightarrow v_{sh} = 0.99995 \, c$!

$$x_{||} = c \, t$$

$\Rightarrow$ particle remains ahead of the shock (=upstream) provided its parallel velocity $c \cos(\theta_{up})$ exceeds $c \beta_{sh}$: $c \cos(\theta_{up}) \geq \beta_{sh} \iff \theta_{up} \leq 1/\gamma_{sh}$

$\Rightarrow$ particle returns downstream once it is deflected by $1/\gamma_{sh}$ on a timescale $t_L/\gamma_{sh}$

$\Rightarrow$ energy gain $\sim \gamma_{b}^2$ at first interaction (isotropic injected population), then $\sim 2$ at subsequent shock crossings...
Test particle Monte Carlo simulations:

assuming isotropic scattering upstream and downstream of the shock front, finds energy gain as predicted...

... combined with a substantial escape probability downstream (∼0.4), leads to a spectral index $s \simeq 2.2-2.3$

(Bednarz & Ostrowski 98, Kirk et al. 00, Achterberg et al. 01, M.L & Pelletier 03, Vietri 03, Nlemiec & Ostrowski 05, Keshet & Waxman 05,...)
so far:

→ in the test particle limit, if scattering is efficient and isotropic, Fermi acceleration is expected to produce powerlaws with index $s \sim 2.3$, acceleration timescale $t_{\text{acc}} \sim t_L / \gamma_{\text{sh}}$

→ however, scattering may neither be efficient nor isotropic...
Some crucial differences w.r.t. non-relativistic Fermi:

→ shock compressed magnetic field is essentially perpendicular in the downstream: isotropy is broken and particles must perform cross-field diffusion to come back to the shock front...

→ the shock moves away from downstream at a large velocity $c/3$, therefore cross-field diffusion must be highly efficient in order for the particle to travel back to the shock front...

→ the general consequence is that relativistic Fermi acceleration is highly inhibited in the test particle limit, unless very intense turbulence (much larger than the mean field) is seeded close to the shock front on very short scales (← to be discussed in the second half)
Ultra-relativistic superluminal shock waves

\[ B_{\perp|sh} = \gamma_{sh} B_{\perp|u} \]
\[ B_{\parallel|sh} = B_{\parallel|u} \]

\( \Rightarrow \) ultra-relativistic shock waves are mostly perpendicular (superluminal)
Relativistic Fermi acceleration - oblique shock waves

- **Ultra-relativistic shock waves are generically superluminal:**
  - the intersection between a magnetic field line and the shock front moves faster than $c$
  - if a particle is tied to a field line, this particle cannot return to the shock front...

- **Scattering from turbulence:**
  - large scale turbulence does not help:
    - the particle cannot execute more than $1\frac{1}{2}$ cycle...
    - up → down → up → down then advection to $-\infty$ (ML et al. 06, Niemiec et al. 06)

large scale: w.r.t. typical Larmor radius (downstream frame) of accelerated particles of Lorentz factor $\gamma_p \sim \gamma_{sh}$

... explanation: the particle is advected on a length scale $c t_{scatt}/3$ away from the shock before it turns around, but $c t_{scatt}/3 \gg l_\perp$, therefore the particle cannot return to the shock...

($t_{scatt}$: scattering timescale in the turbulence, $l_\perp = 3D_\perp/c$ perpendicular diffusion length scale in turbulence)
Relativistic Fermi acceleration - oblique shock waves

- Particle trajectories in shock compressed turbulence:

\[ \Gamma_{sh} = 3.65 \quad \text{and} \quad \Gamma_{sh} = 38.2 \]

- Conditions for successful relativistic Fermi acceleration: (M.L. et al. 06, Pelletier et al. 09)

  \[ \delta B \gg B_0, \quad \ell_c \ll r_L \]

  (in agreement with Monte-Carlo simulations of Niemiec et al. 06)
different physics of acceleration between non-relativistic and relativistic shocks:
relativistic shocks move about as fast as the accelerated particle

→ particle distribution is strongly anisotropic upstream of the shock
→ cross-field diffusion must be highly efficient downstream of the shock

efficient relativistic Fermi acceleration requires a very high level of micro-turbulence,
with $\delta B \gg B_0$ and a typical scale $l_c \lesssim r_L$

note that relativistic shock waves are oblique ($\sim$ perpendicular), unless the angle
between the magnetic field and the shock normal $\lesssim 1/\gamma_{sh}$; the physics
of acceleration in parallel shock waves is different....

modern developments: use PIC simulations to probe the wave-particle relationship
(fully non-linear treatment) and understand the generation and impact of
micro-turbulence on shock dynamics...
of course to be complemented with analytical studies of the nonlinear regime!
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Particle acceleration at unmagnetized shock waves:

observed in PIC simulations: pair plasma, $\gamma_{sh} \sim 20$  (Spitkovsky 08)
PIC simulations of unmagnetized pair shocks

Trajectories of accelerated particles:

Spitkovsky 08
Fermi acceleration without a background magnetic field?

... particles generate the magnetized turbulence that scatters and accelerates these particles, and that mediates the shock transition...
As viewed in the shock frame, the shock transition operates between:

→ far upstream, a cold ion beam and a cold electron beam, the electrons carrying $m_e/m_p$ less kinetic energy than the ions ($\gamma_{sh} m_p c^2$)
→ far downstream, a relativistic plasma of temperature $\sim \gamma_{sh} m_p c^2$, with the electrons at rough equipartition with the ions $\Rightarrow$ Lorentz factor $\gamma_e \sim \gamma_{sh} m_p/m_e$

Reflected ion population: ions with positive velocity in the shock frame at the shock transition, physically reflected by a combination of shock electrostatic barrier and compression of magnetic field....

... as viewed in the upstream frame, corresponds to the first Fermi cycle, hence typical Lorentz factor $\sim \gamma_b^2$

$\Rightarrow$ beam instabilities
Current PIC simulations produce shocks, particle accelerations (in some cases), but do not see evidence for a steady state...

\[ \Rightarrow \text{accelerated particles of higher energy influence the shock structure...} \]
Length and time scales for GRB

**Blast wave geometry**

GRB orders of magnitude:

- **radius for afterglow**: $R \sim 10^{17}$ cm
- **dynamical timescale**: $R/c \sim 10^6$ sec
- **Lorentz factor**: $\gamma_b \sim 100$
- **B field**: $B_{\text{ISM}} \sim 1 \mu$G
- **ISM coherence**: $l_{\text{coh}} \sim 10^{20}$ cm

- **blast width**: $R/\gamma_b^2 \sim 10^{13}$ cm (upstream frame)
- **Larmor**: $r_L \sim 3 \times 10^{16}$ cm (upstream frame)

- **blast width**: $R/\gamma_b \sim 10^{15}$ cm (downstream frame)
- **Larmor**: $r_L \sim 10^{12}$ cm (downstream frame)

- **precursor**: $r_L/(2 \gamma_b^3) \lesssim 10^{10}$ cm (upstream frame)

**PIC scale**: $c/\omega_{\text{pi}} \sim 10^7$ cm

**PIC simulations**: $1000/\omega_{\text{pi}} \sim 0.3$ sec

⇒ **PIC simulations probe a tiny fraction of the evolution of the shock**:

- plasma $\ll$ precursor $\ll$ Larmor, blast $\ll$ ISM coherence
Some current questions

→ Fermi acceleration appears to work in unmagnetized shocks... how does this work for realistic astrophysical shocks of various degrees of magnetization?

→ physics of the instabilities seeded upstream of the shock...

→ how do the instabilities evolve downstream with time and distance?

→ can instabilities be source elsewhere in the downstream?

→ once Fermi acceleration is operational, what is the maximal energy, acceleration timescale...?

→ how do particles radiate and in what kind of turbulence?

→ what can be learned from GRB observations?
Relativistic Fermi acceleration - sources of turbulence

(downstream frame)

external sources of turbulence(?): instability of the blast (Levinson 10), interactions of the shock with external inhomogeneities (Sironi & Goodman 07)...

Fermi acceleration if $L_{\text{growth}} \ll r_L$

Micro-instabilities:

→ for GRB+ISM, micro-instabilities can grow and allow Fermi acceleration,..
  control first cycles of Fermi generations: $\frac{r_L}{(c/\omega_{p_i})} \sim \frac{1}{2} \sqrt{\gamma/\gamma_{\text{min}}}$

...standard GRB model for the afterglow: Fermi acceleration at a relativistic shock wave, with $\varepsilon_B \sim 0.01$ over most of the blast of width $r/\gamma_{\text{sh}} \gg r_L$

→ if magnetization $\gg$ ISM, micro-instabilities are necessary to trigger Fermi cycles in the absence of fast growing instabilities downstream
→ shock reflected and shock accelerated particles move in upstream background field with Lorentz factor $\gamma_{sh}^2$, along shock normal, forming **an unmagnetized beam of Lorentz factor $\gamma_{sh}^2$ and opening angle $1/\gamma_{sh}$**

→ neutral beam instabilities: (e.g. Bret 09)

  
  **Weibel/filamentation** (Gruzinov & Waxman 99, Medvedev & Loeb 99, Lyubarsky & Eichler 06, Wiersma & Achterberg 04, 07, 08; ML & Pelletier 10, 11; Rabinak et al. 10, Shaitsultanov et al. 11)

  **Cerenkov resonance with plasma eigenmodes:** ML & Pelletier 10, 11

  *oblique two stream with electrostatic modes* 
  $$\omega_p = k_x v_{beam}$$

  *Whistler waves* 
  $$\omega_{Wh} = k_x v_{beam}$$

→ charged current instabilities:

  **Buneman mode** (ML & Pelletier 11) ... efficient source of electron heating

  **Bell instability**... for parallel shocks (Bell 04, Reville et al. 06, ML & Pelletier 10)

→ **main limitation:** very short precursor, length $\sim r_{L,0}/\gamma_{sh}^3 \sim \gamma_{sh}^{-1} c/\omega_{ci}$
Magnetization vs shock Lorentz factor...

at high magnetisation, e.m. precursor → wakefield heating /acceleration
e.g. Hoshino et al. 92, Gallant et al. 92, Lyubarsky 06, Hoshino 08

PIC simulations (Sironi & Spitkovsky 11)

too short precursor...
no micro-instabilities...
no Fermi acceleration...

GRB in ISM

ML & Pelletier 10,11

micro-instabilities grow
⇒ Fermi acceleration...

limit for growth of electrostatic mode
limit for growth of filamentation mode

\( \xi \frac{1}{S_{CR}} \gamma_{sh}^2 \sigma_u \sim 1 \)

GRB in ISM

ML & Pelletier 10,11

mildly relativistic shocks...?
Some caveats and generalizations

→ parallel shock waves are not generic in the relativistic regime, but the physics differs and remains open to debate...

→ the previous model ignores:

- the backreaction of higher energy particles (e.g. Keshet et al. 07),
- the evolution of micro-turbulence away from the shock (e.g. Medvedev 05, Keshet et al. 06)
- ...

→ alternative scenarios:

- if external turbulence is sourced at distance $\lesssim l_\perp$ away from the shock, particles can undergo Fermi cycles on this turbulence...

- converter mechanism: if radiative backgrounds are considered, particles can interact and produce neutral particles that are not tied to the magnetic field lines ⇒ modified kinematics, hard powerlaws,... (Derishev et al. 03)
brief summary:

... notwithstanding the previous caveats/generalizations, relativistic Fermi acceleration does not work in the test particle limit...

... but it proceeds efficiently once a realistic shock structure (with feedback of particles on the environment) is considered, provided the Lorentz factor and magnetization are not too high, i.e.

\[ \xi_{cr}^{-1} \gamma_{sh}^2 \sigma_u \sim 1 \]

... indeed, micro-instabilities can grow and scatter the particles as in an unmagnetized shock... good agreement so far with PIC simulations...
Gamma-ray bursts afterglows

**Standard picture:**

→ as the shock propagates, it sweeps up matter from the external medium and dissipates energy through heating in the shock transition:

\[
\frac{dE}{dr} = \gamma_b (\gamma_b - 1) \rho_{\text{ext}} c^2 4\pi r^2
\]

→ beyond radius \( r_{\text{dec}} \sim \left[ \frac{E_{\text{ej}}}{(4\pi \rho_{\text{ext}} c^2 \gamma_b^2)} \right]^{1/8} \) the blast wave decelerates (energy conservation!) with \( \gamma_b \propto (r/r_{\text{dec}})^{-3/2} \) for uniform external density profile

→ electrons are heated to large Lorentz factors \( \gamma_b m_p/m_e \) (downstream frame) and radiate through synchrotron at frequency (observer frame)

\[
\nu_{\text{obs}} \approx 0.2 \frac{eB'}{m_e c} \gamma_b \gamma_e^2 \propto B' \gamma_b^3
\]

→ the photon spectrum is shaped by the electron energy distribution and the cooling efficiency, but the peak frequency moves to lower frequencies as \( \gamma_b \) decreases, and the amount of radiated energy also decreases as \( \gamma_b \) decreases:

→ *decaying afterglow at increasing wavelengths* (\( \gamma \) → X → Opt. → IR → radio...)*
Gamma-ray bursts afterglows

standard afterglow model works "relatively well", suggests efficient acceleration with $\epsilon_e \sim 0.1$, $\epsilon_B \sim 0.01$, in agreement with unmagnetized PIC simulations
GRB afterglows - a lower bound on $B_{\text{upstream}}$

Lower bound on the upstream magnetic field of GRB external shocks: Li & Waxman (06), Li (10)

(1) acceleration timescale $\geq t_{\text{up}} \sim g r_L / \gamma_{\text{sh}} c$, with $g \sim 10$

(2) maximal energy $\gamma_{\text{max}} m_e c^2$ (downstream frame) is obtained by balancing the acceleration timescale with energy loss timescale; if $B_{\text{ext}}$ is small (e.g. ISM), dominant losses are inverse Compton losses on self-produced afterglow synchrotron photons, giving $\gamma_{\text{max}} \propto B_{\text{ext}}^{1/2}$

(3) maximal frequency of synchrotron photons $\nu_{\text{max}} \propto \gamma_{\text{max}}^2 \propto B_{\text{ext}}$

(4) Beppo-SAX afterglows show emission in X as early as 0.1 day, up to 10-50keV:

$$B_{\text{ext}} \gg 200 \left( \frac{n_{\text{ext}}}{1 \text{ cm}^{-3}} \right)^{5/8} \mu\text{G}$$
GRB afterglows - a lower bound on $B_{\text{upstream}}$

- **Evidence for amplification of upstream $B$:**
  - Li & Waxman (06), Li (10): $t_{\text{acc}}$ must be sufficiently short to allow acceleration of particles to energies such that they produce X-ray synchrotron GRB afterglows, $t_{\text{acc}} \propto r_L$ ⇒ lower bound on $B_u$
  - from Beppo-SAX X-ray afterglows as early as 0.1 day, up to 10-50 keV:
    $$B_u \gg 200 \left(n_{\text{u,0}}\right)^{5/8} \mu G$$

- **Implications:**
  - if GRB external shocks propagate in ISM, the external magnetic field must have been "amplified" by at least two orders of magnitude

  however, it is also possible that GRB external shocks propagate in a magnetized environment... e.g. a stellar wind with $B \sim 1000 \text{ G at } 10^{12} \text{ cm, decreasing as } r^{-1}$, $B \sim 10 \text{ mG field at } 10^{17} \text{ cm (observation timescale of } \sim \text{ few hours)}$...

- **What if the circumburst medium is magnetized?**
  - radiative signatures of relativistic Fermi non-acceleration in GRB afterglows?
  - recall: (1) relativistic Fermi acceleration requires small-scale turbulence
    (2) excitation length scale $\sim$ precursor length scale $\sim r_L / \gamma_{\text{sh}}^3$
    (3) therefore, if $B_{\text{ext}}$ too large, the precursor becomes too short, instabilities cannot grow and Fermi acceleration cannot proceed: particles are only shock heated...

  ML & Pelletier 11
Magnetization vs shock Lorentz factor...

at high magnetisation, e.m. precursor $\rightarrow$ wakefield heating /acceleration
e.g. Hoshino et al. 92, Gallant et al. 92, Lyubarsky 06, Hoshino 08

PIC simulations (Sironi & Spitkovsky 11)

mildly relativistic shocks...

$
\begin{array}{c}
\sigma \\
10^{-1} \\
10^{-4} \\
10^{-5}
\end{array}
$

GRB in magnetized wind

too short precursor...
no micro-instabilities...
no Fermi acceleration...

micro-instabilities grow $\Rightarrow$ Fermi acceleration...

$\frac{\varepsilon - 1}{\varepsilon_{cr}} \gamma_{sh}^2 \sigma_u \sim 1$

ML & Pelletier 10,11

PWNe
Early X-ray afterglows - in a magnetized wind

Standard light curve in non-magnetized wind

Recovery: micro-instabilities → Fermi powerlaw

Drop-out: no acceleration + emission of shock heated e shift out of X-ray band

Model:

\[
E = 10^{54} E_{54} \text{ ergs}
\]

\[
\gamma_{ej} = 300 \gamma_{ej,2.5}
\]

\[
T = 10 T_{1} \text{ sec}
\]

\[
B = 1000 \text{G} B_{*}(r/10^{12}\text{cm})^{-1}
\]

\[
\rho = 5 \times 10^{11} A_{*} r^{-2}
\]

\[
t_{d-o} \simeq 110 \text{ sec} E_{54}^{1/3} A_{*}^{-1/3} B_{*}^{2/3} \epsilon_{e,-1}^{4/3} (1+z)^{1/3}
\]

\[
t_{rec} \simeq 10^{4} \text{ sec} E_{54} A_{*}^{-3} B_{*}^{4} \epsilon_{s,-2}^{-2} (1+z)
\]

→ if circumburst medium is magnetized, one expect a clear drop-out in the X-ray range

→ interesting comparison with existing data...
→ relativistic Fermi acceleration differs from non-relativistic Fermi, as the shock wave moves about as fast as the accelerated particles...

→ efficient relativistic Fermi acceleration requires to consider a consistent shock structure with feedback of the accelerated particles on the shock environment

→ current model: relativistic Fermi acceleration works, but at low upstream magnetization, all the more so at large shock Lorentz factors... \[ \xi_{\text{cr}} - \frac{1}{\gamma_{\text{sh}}} \sigma_u \leq 1 \]

→ many open and important questions:

→ spectral index (if any)...
→ source + evolution of turbulence behind the shock...
→ shock evolution on macroscopic timescales...
→ mildly relativistic shocks...
→ more generally, link to observations + real world...
Acceleration timescale at a ultra-relativistic shock

→ for a lower bound on $t_{\text{acc}}$, consider only the motion in upstream: $t_{\text{acc}} > t_u$

→ in shock rest frame, for simple shock drift acceleration, $t_{\text{acc}} \sim r_{L,0}$...

→ if micro-turbulence is added, $r_L \gg l_c$ implies a scattering timescale $t_{\text{scatt}} \sim r_L^2/l_c$

then (Gallant & Achterberg 99, Pelletier et al. 09):

\[ t_u \sim r_L^2/l_c \quad (r_L \ll \frac{\delta B}{B} l_c) \]

\[ A \equiv \frac{t_{\text{acc}}}{t_L} \sim \frac{r_L}{l_c} \gg 1 \]

\[ E_{\text{max}} < \gamma_{sh}^{1/2} c \delta B \sqrt{Rl_c} \]

\[ t_u \sim r_{L,0} \quad (r_L \gg \frac{\delta B}{B} l_c) \]

\[ A \sim \delta B/B \gg 1 \]

\[ E_{\text{max}} < \gamma_{sh} c B_u R \]

→ caveat: isotropic + static turbulence with well defined $l_c$...

... Medvedev & Zakutnyaya (09): $l_c$ increases with distance to shock (but opposite observed in PIC)

... note that in upstream frame, precursor length scale $\sim r_L / \gamma_{sh}^3 \rightarrow$ no gyroresonance
Acceleration – a luminosity bound

- **A generic case:** acceleration in an outflow
  
  - acceleration timescale (comoving frame): \( t_{\text{acc}} = \mathcal{A} t_L \)
  
  \( \mathcal{A} \gtrsim 1, \mathcal{A} \sim 1 \) at most:
    - for non-relativistic Fermi I, \( \mathcal{A} \sim g/\beta_{\text{sh}}^2 \) with \( g \gtrsim 1 \)
  
  - time available for acceleration (comoving frame): \( t_{\text{dyn}} \approx R \frac{R}{\beta \Gamma c} \)
  
  - maximal energy: \( t_{\text{acc}} \leq t_{\text{dyn}} \Rightarrow E_{\text{obs}} \leq \mathcal{A}^{-1} Z e B R / \beta \)
  
  - ‘magnetic luminosity’ of the source: \( L_B = 2 \pi R^2 \Theta^2 \frac{B^2}{8 \pi} \Gamma^2 \beta c \)
  
  - lower bound on total luminosity: \( L_{\text{tot}} \geq 0.65 \times 10^{45} \Theta^2 \Gamma^2 \mathcal{A}^2 \beta^3 Z^{-2} E_{20}^2 \text{erg/s} \)
    
    \( 10^{45} \text{ergs/s is robust:} \)
    
    for \( \beta \rightarrow 0 \), \( \mathcal{A}^2 \beta^3 \geq 1/\beta \geq 1 \)
    
    for \( \Theta \Gamma \rightarrow 0 \), \( L_{\text{tot}} \geq 1.2 \times 10^{45} A \beta \frac{k}{r_{\text{LC}}} Z^{-2} E_{20}^2 \text{erg/s} \)
  
- **Lower limit on luminosity of the source:**
  
  low luminosity AGN: \( L_{\text{bol}} < 10^{45} \) ergs/s
  
  high luminosity AGN: \( L_{\text{bol}} \sim 10^{46}-10^{48} \) ergs/s
  
  gamma-ray bursts: \( L_{\text{bol}} \sim 10^{52} \) ergs/s
  
  \( \Rightarrow \) only most powerful AGN jets, GRBs or magnetars for protons