Formation of relativistic MHD jets: stationary state solutions & numerical simulations

High Energy Phenomena in Relativistic Outflows
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Outline:
1) MHD models of jet formation
2) Stationary state solutions: collimation, acceleration, radiation
3) Numerical simulations: collimation, acceleration (preliminary)

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Astrophysical jets:  
Standard model

**MHD model** of jet formation:

--> **jets** are collimated **disk/stellar winds**, launched, accelerated, collimated by magnetic forces

--> **5 basic questions** of jet theory:

- **collimation & acceleration** of a disk/stellar wind into a jet?
- **ejection** of disk/stellar material into wind?
- **accretion** disk structure?
- **generation of magnetic field**?
- **jet propagation / interaction** with ambient medium
Astrophysical jets: Magnetohydrodynamics (MHD)

- MHD concept: ionized, neutral fluid: average quantities: \( \mathbf{j} = q_e \mathbf{v}_e \rho_e + q_i \mathbf{v}_i \rho_i \)
- Ideal MHD: infinite conductivity, "frozen-in" field lines:
- MHD Lorentz force: \( \mathbf{F}_L \sim \mathbf{j} \times \mathbf{B} \)
- MHD equations (to be solved numerically):

\[
\begin{align*}
\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\rho \left( \partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) + \nabla P + \rho \nabla \Phi &= \mathbf{j} \times \mathbf{B} = 0 \\
\rho \left( \partial_t e + (\mathbf{v} \cdot \nabla) e \right) + P(\nabla \cdot \mathbf{v}) - \eta_D |\mathbf{j}|^2 / c^2 &= 0 \\
\partial_t \mathbf{B} &= \nabla \times (\mathbf{v} \times \mathbf{B} - \eta_D \mathbf{j} / c) \\
\nabla \cdot \mathbf{B} &= 0, \quad \nabla \times \mathbf{B} = 4\pi \mathbf{j} / c
\end{align*}
\]

Axisymmetric flows:
- poloidal, toroidal field components: \( \mathbf{B} = \mathbf{B}_p + \mathbf{B}_\phi \)
- magnetic flux surfaces:

\[ \Psi(R, Z) \sim \int \mathbf{B}_p \cdot d\mathbf{A} \]

Lorentz force components (1)

- (de/) accelerating:

\[ \mathbf{F}_{L,\parallel} \equiv \mathbf{j}_\perp \times \mathbf{B}_\phi \]

- (de-) collimating:

\[ \mathbf{F}_{L,\perp} \equiv \mathbf{j}_\parallel \times \mathbf{B} \]
Astrophysical jets: Magnetohydrodynamics (MHD)

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  \rho (\partial_t e + (\vec{v} \cdot \nabla) e) + P (\nabla \cdot \vec{v}) - \eta_D |\vec{j}|^2 / c^2 &= 0 \\
  \partial_t \vec{B} &= \nabla \times (\vec{v} \times \vec{B} - \eta_D \vec{j} / c) \\
  \nabla \cdot \vec{B} &= 0, \quad \nabla \times \vec{B} = 4\pi \vec{j} / c
  \end{align*}
  \]

  Lorentz force components (2):
  
  \[
  \vec{F}_L = \nabla \left( \frac{|\vec{B}|^2}{8\pi} \right) + \frac{1}{4\pi} \left( \vec{B} \cdot \nabla \right) \vec{B}
  \]

  \( \rightarrow \) (de-) accelerating, (de-) collimating

  \( \rightarrow \) e.g.: pure dipole is force-free: \( \vec{F}_L = 0 \)
MHD jet formation: Acceleration & collimation

Magneto-centrifugal acceleration: (Blandford & Payne 1982)
-> field lines corotate w/ disk, "beads on wire"
-> strong poloidal field
-> field line inclination < 60 deg
  --> unstable equilibrium, centrifugal sling-shot

Magnetic pressure acceleration: (Uchida & Shibata 1984, Contopoulos 1994)
-> coupling of differentially rotating regions
-> poloidal field twisted -- toroidal field
  --> weak poloidal field, strong toroidal field
  --> magnetic pressure gradient
  --> inflation, material expelled in axial direction

MHD acceleration:
-> Lorentz force ~ $j \times B_\phi$

Self-collimation of MHD jets:
Alfven radius:
  where kinetic ~ magnetic energy:
-> poloidal field twisted by inertia
-> collimation by toroidal field tension
MHD jet formation: 
Relativistic jets

Relativistic effects:

-> relativistic Alfven surface: \(0 = 1 - M_A^2 - (R_A / R_L)^2\); \(M_A^2 = \frac{4 \pi \mu n u_p^2}{B_p^2}\); \(u_p = \gamma v_p\)

-> light cylinder \(\sim\) Alfven surface (not self-similar)
\[\Omega_F (\Psi) R_L (\Psi) = c \quad \left(\text{for } SS \rightarrow \Omega_F (\Psi) \sim R^{-1} \rightarrow v_\phi (r) \neq v_K (r)\right)\]

-> electric field: de/collimating force:
\[E = -\frac{1}{c} \Omega_F \nabla \Psi; \quad E_{\text{perp}} = \frac{R}{R_L} B_p\]
\[F_{\text{perp}} = \ldots + \left(1 - \frac{R^2 \Omega_F^2}{c^2}\right) \nabla_{\text{perp}} \frac{B_p^2}{8 \pi} + \ldots\]

-> check: is MHD condition satisfied: particle density \(\gg\) charge density:
\[n \gg n_{GJ} = \frac{- (\nabla \Psi \cdot \nabla) (r^2 \Omega_F)}{4 \pi c e r^2} = \frac{B_z (r; \Psi)}{2 \pi c e R_L}\]

-> BH: Kerr geometry (not self-similar)
\[ds^2 = \alpha^2 c^2 dt^2 - \tilde{\omega}^2 (d\phi - \omega dt)^2 - (\rho^2 / \Delta) dr^2 - \rho^2 d\theta^2\]
redshift \(\alpha\)
frame dragging \(\omega\)
\[\rho^2 \equiv r^2 + a^2 \cos^2 \theta \quad \Delta \equiv r^2 - 2 GM r / c^2 + a^2\]
\[\Sigma^2 \equiv (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta \quad \tilde{\omega} \equiv (\Sigma / \rho) \sin \theta\]
\[\omega \equiv 2 a GM r / c \Sigma^2 \quad \alpha \equiv \rho \sqrt{\Delta / \Sigma}\]
Stationary relativistic MHD jets

Relativistic jet magnetospheres

Solutions of stationary relativistic MHD equations:

-> jet structure: force-free (strong field): Grad-Shafranov equation (force-balance across field)

\[ \tilde{\omega} \nabla \cdot \left( \alpha \frac{D}{\tilde{\omega}^2} \nabla \Psi \right) = \tilde{\omega} \left( \frac{\omega - \Omega_F}{\alpha c^2} \Omega'_F \right) \nabla |\nabla \Psi|^2 - \frac{1}{\alpha} \frac{4}{c^2} I I' \]

\[ D = 1 - \left( \frac{\tilde{\omega}}{\tilde{\omega}_L} \right)^2 \]

-> jet acceleration: wind equation along field line, considers inertial terms (force-balance along the field)

--> rewrite energy conservation as polynomial for poloidal velocity

\[ u_p^2 + 1 = -\sigma_m \left( \frac{E}{\mu} \right)^2 \frac{k_0 k_2 + \sigma_m 2 k_2 M^2 - k_4 M^4}{(k_0 + \sigma_m M^2)^2} \]

\[ k_0 = g_{33} \Omega_F^2 + 2 g_{03} \Omega_F + g_{00}, \]

\[ k_2 = 1 - \Omega_F \frac{L}{E}, \]

\[ k_4 = - \left( g_{33} + 2 g_{03} \frac{L}{E} + g_{00} \frac{L^2}{E^2} \right) / (g_{03} - g_{00} g_{33}) \]

--> truly 2.5 D force-balance --> global structure & dynamics
Global jet/field structure:

Example 1: solution of GR force-free GS equation:

--> general relativistic features:
  -> gravitational redshift, time lapse
  -> frame dragging: “rotation of space”
  -> light “cylinder”: “rotation” of magnetosphere

--> results (Fendt 1997, finite element code):
  -> true 2.5 D force-balance across the field
  -> global structure of the jet, R_jet > 1000 R_g
  -> BH, inner light cylinder resolved; defines b.c.
  -> asymptotic solution exactly matches special relativistic 1D jet structure
  -> electric current I(Ψ)~RBφ conserved along Ψ
    -> helical field B_p + Bφφ
  -> rapid collimation:
    -> collimating jet edge defined by internal local force-balance (regularity condition)

--> limitations: R < 10 R_L (??), force-free asymptotics (??)
Stationary relativistic MHD jets

**Relativistic jet magnetospheres**

**Global jet/field structure:**

Example 2: solution of SR force-free GS equation:
- light “cylinder”: “rotation” of magnetosphere incl. differential rotation \( \Omega(\Psi) \)

--> results (Fendt & Memola 2001):
- iterative search for light surface as \( \Omega(\Psi) = \Omega(\Psi(r,z)) = \Omega(r,z) \)
- asymptotic jet consistent with special relativistic 1D solution (electric current)
- rapid collimation:
  - compare to M87 VLBI data (Junor et al. 99):
    - jet formed within 30 Rs
    - initial 60° opening angle
    - strong collimation outside 0.04 pc
    - jet radius 120 Rs, light cylinder ~50 Rs

Example 3:
GR solution incl. diff. rotation (not yet solved)
- link between asymptotic jet & accretion disk
Stationary relativistic MHD jets

Relativistic jet magnetospheres

Flow dynamics along field lines:

Example 4: Ultrarelativistic jets of GRBs

-> solve "cold" wind equation along given field line -> collimating field structure prescribed:

-> compare different field distribution

\[ \Phi \equiv B_p r^2 = \Phi(r; \Psi) \equiv r^{-q} \]

-> compare different mass load, magnetisation

-> modified Michel scaling:

\[ \Gamma_{\infty} \sim \sigma^{1/3} \]

-> high magnetisation

\[ \sigma \sim 1000-5000 \]

\[ u_{p,\infty} \]

-> ultra-relativistic speed

\[ \Gamma \sim 1000 \]

-> asymptotic energy distribution:

\[ q \sim 0 \text{ stays Poynting dominated} \]

\[ q > 0 \text{ } \rightarrow \text{ equipartition} \]

(see Begelman & Li 94)

\[ q = 0.01, 0.1, 0.2 \]

Fendt & Ouyed 2004
Stationary relativistic MHD jets

Relativistic jet magnetospheres

Flow dynamics along field lines:

Example 4: Ultrarelativistic jets of GRBs (Fendt & Ouyed 2004)

magnetisation $\sigma < 5000$ -> check for MHD condition a posteriori:

$n \gg n_{GJ} = \frac{B_z(r; \Psi)}{2\pi ce RL}$

- ok for the wind solutions derived

-> for large radii:

$n/n_{GJ} \sim 100 \sim$ constant:

since collimation ($q = 0.1, 0.2$)
Stationary relativistic MHD jets

Relativistic jet magnetospheres

Flow dynamics along field lines:

Example 4: Ultrarelativistic jets of GRBs

- “bottle-neck” instability for re-collimating jets:
  - if $\Phi(r;\Psi)$ decreases at large radii:
  - no stationary solution at re-collimation point
  - onset for shock solution triggering GRB?

- Note: re-collimation of flux tube natural for transition from radial to cylindrical field structure

Fendt & Ouyed 2004
Stationary relativistic MHD jets

Radiation from relativistic jets

**Aim:** get radiative signature from MHD solution:
- e.g. keV-spectrum from micro-quasar
  --> accretion disk $T < 10^9$ (close to center of gravity)
  --> hot disk material is loaded into jet

**1st step:**
- stationary state MHD dynamics of the accelerating jet
  - e.g. Kerr metric, hot wind equation
  - spatial scale: footpoint $\sim 5 \, R_g$ to $500 \, R_g$
  - find critical solution
    - get MHD variables: $\rho(s)$, $u_p(s)$, $u_\phi(s)$, $T(s)$

\[ u_p(r;z) \]
\[ \rho(r;z) \]
Stationary relativistic MHD jets

Radiation from relativistic jets

2nd step:
-> spectrum for each of (e.g. 5000) fluid element \((v, T, \rho)\) along collimating jet cone
-> thermal radiation (Bremsstrahlung)
-> continuum dominant for \(T > 10^9\) K
-> O,N,Fe,Ne,S... lines (0.5-0.9 KeV), Ne,Fe,Mg,Ni ...(1-4 KeV) FeXXV, FeXXVI emission (6.6-7.0 KeV)

-> \(L_X\) of jet tori (\(\dot{m}M \sim 10^{-10}\) Msun/yr, \(M^* = 5\) Msun):

\[10^6.64\) K\]
\[10^7\) K\]
\[10^8\) K\]
Stationary relativistic MHD jets

Radiation from relativistic jets

3rd step: combine spectrum from all 5000 fluid elements for certain l. o. s. -> beaming, shift with respect to l. o. s. -> total X-luminosity:

4 x 10^31 ergs/s (rest frame)
6 x 10^32 ergs/s (along axis)
2 x 10^33 ergs/s (20° inclination)
<< L_kin ~ 10^39 ergs/s

Memola et al. (2002)
Simulations of MHD jet formation

Newtonian disk jets


Model assumptions (OP 97):
- ideal MHD
- Keplerian disk as boundary condition, prescribed mass flow rate
- disk magnetosphere, Keplerian footpoints
- mass injection from disk, inner disk radius, polytropic gas + turb. Alfvénic pressure
- advantage: numerical stability
  - to follow evolution over 1000s of rotation periods
  - to find stationary state solution (if existent)
Collimation of disk jets:

--> model setup: pure disk wind as boundary condition (no star), disk potential magnetic field

--> collimation degree quantified of outflow by mass flux in axial versus radial direction

--> proof of MHD jet self-collimation under variation of boundary conditions

--> effect of turbulent magnetic diffusivity --> decollimation (Fendt & Cemeljic 2002)

--> variation of the mass injection / disk magnetic field profiles (Fendt 2006)

Example simulation:  

\[ \delta_i = 100, \ \beta_p = \beta_\phi = 1, \ \beta_T = 0.03, \ v_{\text{inj}}(r) = 10^{-3} v_K(r), \ \rho_{\text{inj}} = 100 \ \rho_{\text{cor}}, \ r_{\text{max}} = 40, \ z_{\text{max}} = 160 \]
Main goal: check acceleration & collimation of relativistic MHD jets:

--> model setup: pure disk wind as boundary condition;
    initial potential magnetic field & hydrostatic corona;
    axisymmetry; polytropic gas $\gamma = 4/3$

--> disk structure not treated in simulation (compare to
    Hawley & DeVillier etal, Nishikawa etal, McKinney etal)

--> allows for parameter study for field distribution,
    mass load/magnetisation distribution etc

--> relativistic MHD code PLUTO (Mignone et al. 2006)

--> Newtonian gravity

--> aim: low plasma-$\beta$, highly magnetized flows

--> $\delta_i = 3.5$, plasma-$\beta_i = 0.1$, $\eta_i = 1.0$, $h_{\text{disk}} = 0.2$, $z_g = 0.2$, $v_{\text{inj}} = 0.01 \, c$, $v_{k{\text{in}}} = 0.4 \, c$

--> scaled grid in R,Z (non-equidistant): 0.0 to 1.0 $R_{\text{in}}$ 60 (equidistant) elements

1.0 to 40.0 $R_{\text{in}}$ 200 (non-eq.-dist) elements

\begin{align*}
  i &= 50 \quad 60 \quad 100 \quad 150 \quad 200 \quad 260 \\
  R &= 0.82 \quad 1.0 \quad 2.07 \quad 5.21 \quad 13.1 \quad 39.6
\end{align*}
MHD jet collimation:  
Disk jets (relativistic jets)

Preliminary results:

--> time evolution of jet formation for
   ~ 100 inner disk rotations
   ~ 0.5 outer disk rotations

--> acceleration of slow disk wind:
   maximum speed ~0.8 c
   note: 0.4c Keplerian disk,
   mass injection with 0.01x0.4c

--> two components:
  -> narrow high speed funnel
     (reaches stationary state)
  -> slower disk wind (still evolving)

--> collimation of high speed flow,
   opening angles < 10°

(Fendt et al. 2007)
MHD jet collimation:
Disk jets (relativistic jets)

Preliminary results:

--> MHD structure:
  density distribution:
  hydro-static corona --> disk outflow
  --> steady corona along axis
  (--> BZ jet from BH ??)

negative $v_z(r,z)$

\[
\begin{align*}
\text{TIME} & \quad 0.00 & \quad \rho(r,z) \\
\text{TIME} & \quad 200.00 & \quad \rho(r,z)
\end{align*}
\]
**MHD jet collimation:**

**Disk jets (relativistic jets)**

**Preliminary results:**

--> MHD structure:

\[ \text{collimation: opening angle } \alpha \text{ defined by mass flux} \]

\[ \text{at } j= 0, 100, 200, z= 0.5, 2.1, 14.0, t=210 \]

--> opening angle of high speed beam < 5-10°

--> since sub-Alfvenic domain:

\[ \text{collimation by disk wind?} \]

\[ \alpha(z=0), \text{magnetic field} \]

\[ \alpha(z=0.5) \]

\[ \alpha(z=2) \]

\[ \alpha(z=15) \]
(1) Common model: astrophysical jets are collimated MHD disk winds (matter content)
   Stellar jets: intrinsic stellar wind component
   Relativistic jets: Blandford-Znajek jet component

(2) **Stationary state** MHD solutions (critical surfaces) may provide global solutions
    for jet structure and dynamics, however, yet to be found.
    **Limitations:** self-similarity, force-freeness, non-local force balance ...
    Relativistic nature hardly treated by self-similar approach, however self-consistent
    treatment of inertial terms.

(3) Example solutions (Fendt et al):
    -> **high Lorentz factors** reached for highly magnetized flows; asymptotic flow in
       E equipartition; MHD wind dynamics used to derived **radiative features**
    -> **rapid collimation** of 2.5 D field structure (helical field)
    -> “global” solutions from Rs to Rjet ~1000-10000 Rs

(4) Newtonian **MHD simulations** of disk jets confirm **MHD self-collimation** (Ouyed & Pudritz)

(5) Simulations including the disk structure may provide outflow mass loss rates,
    **times scales** of ejection process.
    However, long-term simulations are essential:
    -> sufficiently stable disk models are required & proper treatment of BH b.c.
    Substantial progress made recently (Hawley et al, Nishikawa et al, McKinney et al, ...).

(6) Preliminary simulations (Fendt et al): axisymmetric MHD, PLUTO, fixed disk b.c.:
    -> **collimated fast beam,** possibly collimated by surrounding disk wind
    -> **low plasma-β** --> substantial increase in outflow speed (0.4 c disk --> 0.8 c jet)
    -> **parameter studies** for jet launching parameters

**Summary**