Magnetorotational supernovae, magnetorotational instability

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Outline

• Magnetorotational (MR) mechanism of supernova explosion.
• Core-collapse simulations.
• MR supernova with quadrupole field.
• Magnetorotational instability (MRI) in MR supernova.
• MR supernova with dipole field, jet formation.
• MR supernova – different core masses
• Mirror symmetry violation of the magnetic field in rotating stars – one-sided jets, kicks.
• Conclusions.
Magnetorotational mechanism for the supernova explosion
Bisnovatyi-Kogan (1970)

Amplification of magnetic fields due to differential rotation, angular momentum transfer by magnetic field. Part of the rotational energy is transformed to the energy of explosion

First 2D calculations: LeBlanc & Wilson (1970) -> too large initial initial magnetic fields. \( E_{\text{mag0}} \approx E_{\text{grav}} \Rightarrow \text{axial jet} \)


It is popular now!

The realistic values of the magnetic field are: \( E_{\text{mag}} \ll E_{\text{grav}} \) (\( E_{\text{mag}}/E_{\text{grav}} = 10^{-8}-10^{-12} \))

Small initial magnetic field - is the main difficulty for the numerical simulations.

The hydrodynamical time scale is much smaller than the magnetic field amplification time scale \((\text{if magnetorotational instability is neglected})\).

Explicit difference schemes can not be applied. (CFL restriction on the time-step). Implicit schemes should be used.
Basic equations: MHD + self-gravitation, infinite conductivity:

\[
\frac{d\mathbf{x}}{dt} = \mathbf{u}, \quad \frac{d\rho}{dt} + \rho \text{div}\mathbf{u} = 0,
\]

\[
\rho \frac{du}{dt} = -\text{grad}\left( p + \frac{\mathbf{H} \times \mathbf{H}}{8\pi} \right) + \frac{1}{4\pi} \text{div}(\mathbf{H} \otimes \mathbf{H}) - \rho \text{grad}\Phi
\]

\[
\rho \frac{d\varepsilon}{dt} + p\text{div}\mathbf{u} + \rho F(\rho, T) = 0, \quad p = P(\rho, T), \varepsilon = E(\rho, T),
\]

\[
\Delta\Phi = 4\pi G\rho,
\]

\[
\rho \frac{d}{dt} \left( \frac{\mathbf{H}}{\rho} \right) = \mathbf{H} \cdot \nabla\mathbf{u}.
\]

Additional condition div\mathbf{H} = 0

Axis symmetry \( \frac{\partial}{\partial \phi} = 0 \) and equatorial symmetry \( z=0 \) are supposed.

Notations:

\[
\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u} \times \nabla, \quad \mathbf{x} = (r, \phi, z), \quad \mathbf{u} - \text{velocity, } \rho - \text{density, } p - \text{pressure},
\]

\( \mathbf{H} - \text{magnetic field, } \Phi - \text{gravitational potential, } \varepsilon - \text{internal energy,} \)

\( G - \text{gravitational constant, } R - \text{gas constant, } \gamma - \text{adiabatic index}. \)
Boundary conditions

Axial symmetry

\[ r = 0 : u_r = u_\phi = H_r = H_\phi = \text{rot}_r \mathbf{H} = \text{rot}_\phi \mathbf{H} = 0, \]

Equatorial symmetry

Quadrupole field

\[ u_z = H_z = 0, \]

Dipole field

\[ z = 0 : \quad \text{or} \quad u_z = \frac{\partial B_z}{\partial z} = 0, \]

Outer boundary: \( P = \rho = T = H_\phi = 0, \mathbf{H}_{\text{poloidal}} = \mathbf{H}_q \)

(from Biot-Savarat law)
Difference scheme

Lagrangian, implicit, triangular grid with rezoning,
Complete conservation=>angular momentum conserved automatically

Grid reconstruction

*Elementary reconstruction:* BD connection is introduced instead of AC connection. The total number of the knots and the cells in the grid is not changed.

*Addition a knot at the middle of the connection:* the knot E is added to the existing knots ABCD on the middle of the BD connection, 2 connections AE and EC appear and the total number of cells is increased by 2 cells.

*Removal a knot:* the knot E is removed from the grid and the total number of the cells is decreased by 2 cells.
Difference scheme

Method of basic operators (Samarskii) – grid analogs of basic differential operators:

- $\text{GRAD}(\text{scalar})$ (differential) \sim $\text{GRAD}(\text{scalar})$ (grid analog)
- $\text{DIV}(\text{vector})$ (differential) \sim $\text{DIV}(\text{vector})$ (grid analog)
- $\text{CURL}(\text{vector})$ (differential) \sim $\text{CURL}(\text{vector})$ (grid analog)
- $\text{GRAD}(\text{vector})$ (differential) \sim $\text{GRAD}(\text{vector})$ (grid analog)
- $\text{DIV}(\text{tensor})$ (differential) \sim $\text{DIV}(\text{tensor})$ (grid analog)

Implicit scheme. Time step restrictions are weaker for implicit schemes.

The scheme is completely conservative:
Conservation of the mass, momentum and energy.

The scheme is Lagrangian=> conservation of angular momentum.
Numerical method testing

The method was tested on the following tests:

1. Collapse of a dust cloud without pressure
2. Decomposition of discontinuity problem
3. Spherical stationary solution with force-free magnetic field,
4. MHD piston problem,
Example of the triangular grid
Presupernova Core Collapse

Equations of state take into account degeneracy of electrons and neutrons, relativity for the electrons, nuclear transitions and nuclear interactions. Temperature effects were taken into account approximately by the addition of radiation pressure and an ideal gas.

Neutrino losses were taken into account in the energy equations.

A cool white dwarf was considered at the stability limit with a mass equal to the Chandrasekhar limit.

To obtain the collapse we increase the density at each point by 20% and we also impart uniform rotation on it.
Equations of state (approximation of tables)

\[ P = P(\rho, T) = P_0(\rho) + \mathcal{R} T \rho + \frac{T^4 \sigma}{3} \]

\[ P_0(\rho) = \begin{cases} 
    P_0^{(1)} = b_1 \rho^{5/3} (1 + c_1 \rho^{1/3}), & \text{for } \rho \leq \rho_1 \\
    P_0^{(k)} = a k b_1^{1/k} (\rho - 8.419)^{c_k}, & \text{for } \rho_{(k-1)} \leq \rho \leq \rho_k, \quad k = 2, 6. 
\end{cases} \]

\[ \varepsilon = \varepsilon(\rho, T) = \varepsilon_0(\rho) + \frac{3}{2} \mathcal{R} T + \frac{\sigma T^4}{\rho} + \varepsilon_{Fe}(\rho, T), \quad \varepsilon_{Fe}(\rho, T) = \frac{E_{b,Fe}}{A_m} \left( \frac{T - T_{0,Fe}}{T_{1,Fe} - T_{0,Fe}} \right) \]

Neutrino losses: URCA processes, pair annihilation, photo production of neutrino, plasma neutrino

**URCA:**

\[ f(\rho, T) = 1.3 \times 10^9 \tau(T) /[1 + (7.1 \times 10^{-5} \rho / T^3)^{2/5}] \text{erg s}^{-1} \text{cm}^{-1} \]

\[ \tau(T) = \begin{cases} 
    1, \quad T < 7, \\
    664.31 + 51.024(T - 20), \quad 7 \leq T \leq 20, \\
    664.31, \quad T > 20, 
\end{cases} \]

Approximation of tables from Ivanova, Imshennik, Nadyozhin, 1969

\[ F(\rho, T) = f(\rho, T)e^{-\tau_v} \quad \text{neutrino diffusion} \]
**Initial state**

\[ M = 1.2042 \times M_{\text{sun}} \], spherically symmetrical stationary state, initial angular velocity 2.519 \,(1/\text{sec})

Initial temperature distribution \[ T = \delta \rho^{2/3} \]
Maximal compression state

Max. density $= 2.5 \cdot 10^{14} \text{g/cm}^3$
Neutron star formation in the center and formation of the shock wave

TIME = 4.12450792 (0.14246372 sec)
Mixing
Bounce shock wave does not produce SN explosion :(
Angular velocity (central part of the computational domain). Rotation is differential.
The period of rotation of the young neutron star is about 0.001-0.003 sec

Rapidly rotating pre-SN Crab pulsar P=33ms – rapid rotation at birth

Core collapse in binaries
Initial magnetic field

\[ H = \frac{1}{c} \int \frac{\mathbf{J} \times \mathbf{R}}{R^3} \, dv, \]

Biot-Savart law

\[ J_\phi \rightarrow (H_r, H_z) \]
Initial magnetic field – quadrupole-like symmetry

Toroidal magnetic field amplification.

pink – maximum_1 of $Hf^2$          blue – maximum_2 of $Hf^2$
Maximal values of $Hf = 2.5 \cdot 10^{16}$G

After SN explosion at the border of neutron star $H = 2 \cdot 10^{14}$G
Temperature and velocity field
Specific angular momentum $rV_\phi$
Time evolution of the energies
Time evolution of the energies

Gravitational energy

Internal energy

\[ E_{\text{grav}} \]

\[ E_{\text{int}} \]
Time evolution of the energies

Neutrino losses (ergs)

Neutrino luminosity (ergs/sec)
Ejected energy and mass

Ejected energy \(0.6 \times 10^{51} \text{erg}\)  
Ejected mass \(0.14 M_\odot\)

Particle is considered “ejected” – if its kinetic energy is greater than its potential energy.
Magnetorotational supernova in 1D
Ardeljan et al. 1979

\[ t_{\text{explosion}} : \frac{1}{\sqrt{\alpha}}, \quad \left( \alpha = \frac{E_{\text{mag}0}}{E_{\text{grav}0}} \right) \]

Example: \( \alpha = 10^{-2} \Rightarrow t_{\text{explosion}} = 10, \)

\( \alpha = 10^{-12} \Rightarrow t_{\text{explosion}} = 10^6 \)
Magnetorotational explosion for the different $\alpha = \frac{E_{\text{mag}0}}{E_{\text{grav}0}} = 10^{-2} - 10^{-12}$

Dependence of the explosion time from $\alpha = \frac{E_{\text{mag}0}}{E_{\text{grav}0}}$

$t_{\text{explosion}} \sim -\log_{10}(\alpha)$ (for small $\alpha$)

Example:

$\alpha = 10^{-6} \Rightarrow t_{\text{explosion}} \sim 6$,

$\alpha = 10^{-12} \Rightarrow t_{\text{explosion}} \sim 12$. 
Toy model for MRI in the magnetorotational supernova

\[ \frac{dH_\varphi}{dt} = H_r \left( r \frac{d\Omega}{dr} \right); \quad \text{at the initial stage of the process } H_\varphi < H_\varphi^* : H_r \left( r \frac{d\Omega}{dr} \right) = \text{const}, \]

beginning of the MRI => formation of multiple \textit{poloidal} differentially rotating vortexes

\[ \frac{dH_r}{dt} = H_{r0} \left( \frac{d\omega_\varphi}{dl} \right), \quad \text{in general we may approximate: } \left( \frac{d\omega_\varphi}{dl} \right) \approx \alpha (H_\varphi - H_\varphi^*). \]

Assuming for the simplicity that \( r \frac{d\Omega}{dr} \) = \( A \) is a constant during the first stages of MRI, and taking \( H_\varphi^* \) as a constant we come to the following equation:

\[ \frac{d^2}{dt^2} (H_\varphi - H_\varphi^*) = A H_{r0} \alpha (H_\varphi - H_\varphi^*) \]

\[ \downarrow \]

\[ \begin{align*}
H_\varphi &= H_\varphi^* + H_{r0} e^{\sqrt{A\alpha} H_{r0} (t-t^*)}, \\
H_r &= H_{r0} + \frac{H_{r0}^3}{\sqrt{A}} \alpha^{1/2} \left( e^{\sqrt{A\alpha} H_{r0} (t-t^*)} - 1 \right)
\end{align*} \]
Magnetorotational instability

Central part of the computational domain. Formation of the MRI.
Initial magnetic field – dipole-like symmetry
Magnetorotational explosion for the dipole-like magnetic field
Magnetorotational explosion for the dipole-like magnetic field
Magnetorotational explosion for the dipole-like magnetic field
Ejected energy and mass (dipole)

Ejected energy $\approx 0.5 \cdot 10^{51}$ erg \hspace{1cm} Ejected mass $\approx 0.14M_\odot$

Particle is considered “ejected” – if its kinetic energy is greater than its potential energy

![Graph showing energy and mass over time]
MR supernova – different core masses

Bisnovatyi-Kogan, SM, 2007 (in preparation)

Dependence of the MR supernova explosion energy on the core mass

![Graph showing the dependence of MR supernova explosion energy on core mass](image)
Characteristic time of the magnetic field reconnection

Petcheck mechanism – characteristic reconnection time

\[ \tau_{\text{reconn}} = \frac{4(\ln(\text{Re}_m) + 0.74)}{\pi v_A l^{-1}} \]

Our estimations show:
- Conductivity \( \sim 8 \times 10^{20} \text{c}^{-1} \)
- Magnetic Reynolds number \( \sim 10^{15} \)

Characteristic time of the magnetic field reconnection
For the magnetorotational supernova is:
\[ \tau_{\text{reconn}} \approx 5 \text{c} \]

(approximately 10 times larger than characteristic time of magnetorotational supernova explosion).

Reconnection of the magnetic field does not influence significantly on the supernova explosion.
Mirror symmetry violation of the magnetic field in rotating stars

- a. Initial toroidal field
- b. Initial dipole field
- c. Generated toroidal field
- d. Resulting toroidal field
Mirror symmetry violation of the magnetic field in rotating stars

Resulting toroidal field is larger in the upper hemisphere.

Violation of mirror symmetry of the magnetic field in magnetorotational explosion leads to: One sided ejections along the rotational axis

Rapidly moving radiopulsars (up to 300 km/s).
In reality we have dipole + quadrupole + other multipoles…

(Lovelace et al. 1992)

Dipole \sim \frac{1}{r^3}

Quadrupole \sim \frac{1}{r^4}

The magnetorotational supernova explosion is always asymmetrical.

At high magnetic fields neutrino cross-section depends on the magnetic field value.

The pulsar kick velocity can be up to 1000 km/s along rotational axis
(Bisnovatyi-Kogan 1993)

Jet, kick and axis of rotation are aligned in MR supernovae.
Evidence for alignment of the rotation and velocity vectors in pulsars

“We present strong observational evidence for a relationship between the direction of a pulsar's motion and its rotation axis. We show carefully calibrated polarization data for 25 pulsars, 20 of which display linearly polarized emission from the pulse longitude at closest approach to the magnetic pole…” we conclude that the velocity vector and the rotation axis are aligned at birth.

Rapidly moving pulsar VLBI observations

Determination of pulsar parallaxes and proper motions addresses fundamental astrophysical questions. We have recently finished a VLBI astrometry project to determine the proper motions and parallaxes of 27 pulsars, thereby doubling the total number of pulsar parallaxes. Here we summarize our astrometric technique and present the discovery of a pulsar moving in excess of 1000 kms, PSR B1508+55.

W.H.T. Vlemmings et al. MmSAI, 2005, 76, 531
Cassiopea A- supernova with jets-an example of the magnetorotational supernova 
Conclusions

1. Magnetorotational mechanism (MRM) produces enough energy for the core collapse supernova.

2. The MRM is weakly sensitive to the neutrino cooling mechanism.

3. MR supernova shape depends on the configuration of the magnetic field and is always asymmetrical.

4. MR supernova energy depends on the core mass.

5. MRI appears in MR supernova explosion.

6. One sided jets and rapidly moving pulsars can appear due to MR supernovae.